$$\underset{\text{fast}}{e^-} + \underset{\text{slow}}{\gamma} \longrightarrow \underset{\text{slow}}{e^-} + \underset{\text{fast}}{\gamma}$$

Inverse Compton scattering

Lake side discussion 7.7.21

Inverse ?

E_{el} / ΓE_γ	"standard" $\beta\Gamma \ll 1$	"inverse" $\beta\Gamma \gg 1$	
$\Gamma E_\gamma \ll mc^2$	"standard" Compton scattering e^- at rest, γ bounces off, e^- at rest	Thomson	"Thomson"
$\Gamma E_\gamma \gg mc^2$	KN "standard" Compton scattering e^- at rest, γ scatters, both move	Klein-Nishina	"Klein-Nishina"

δ approximation

→ talk about "monoenergetic isotropic particle" ⇒ electron photon

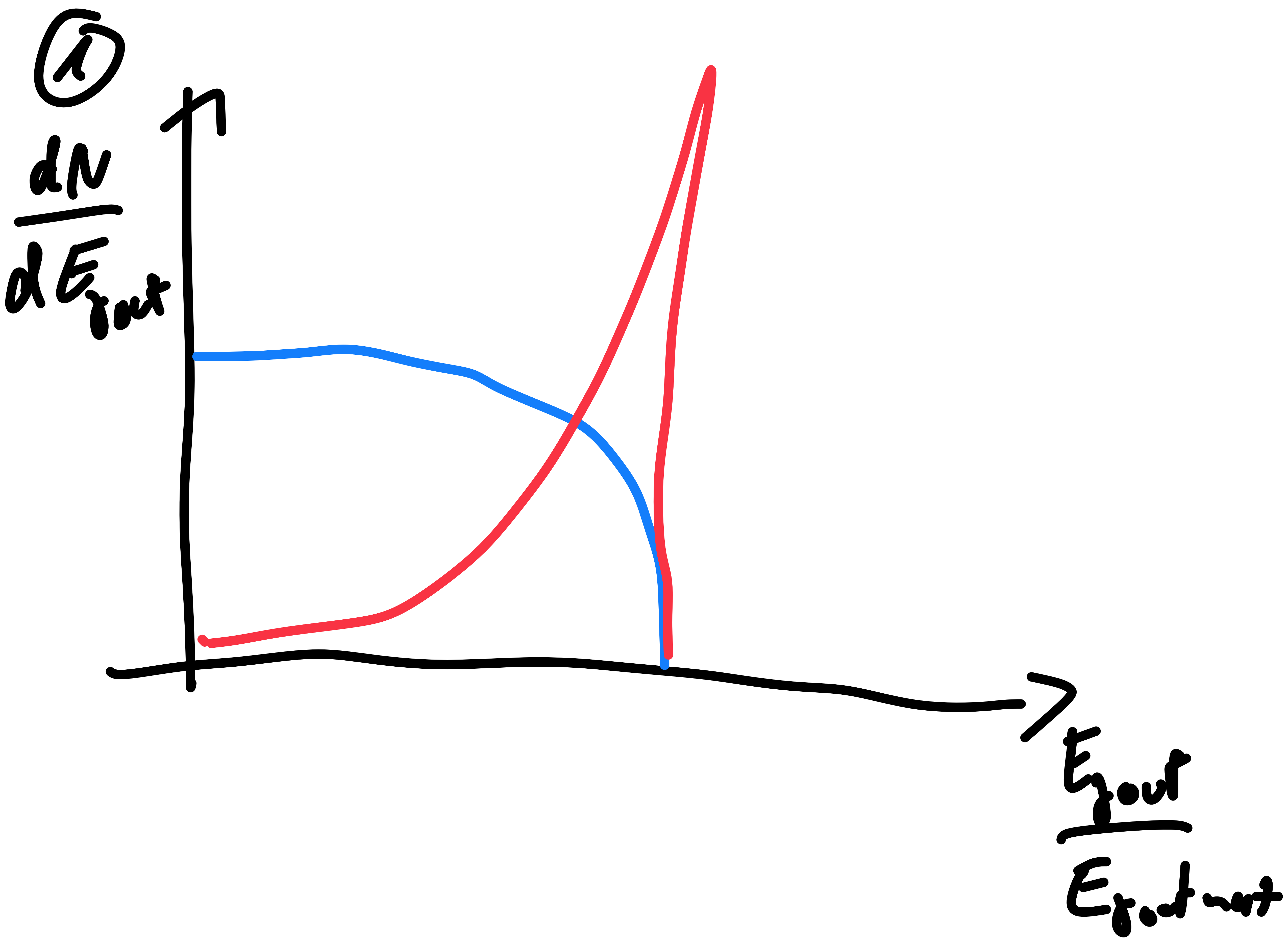
monoenergetic ↓ single E

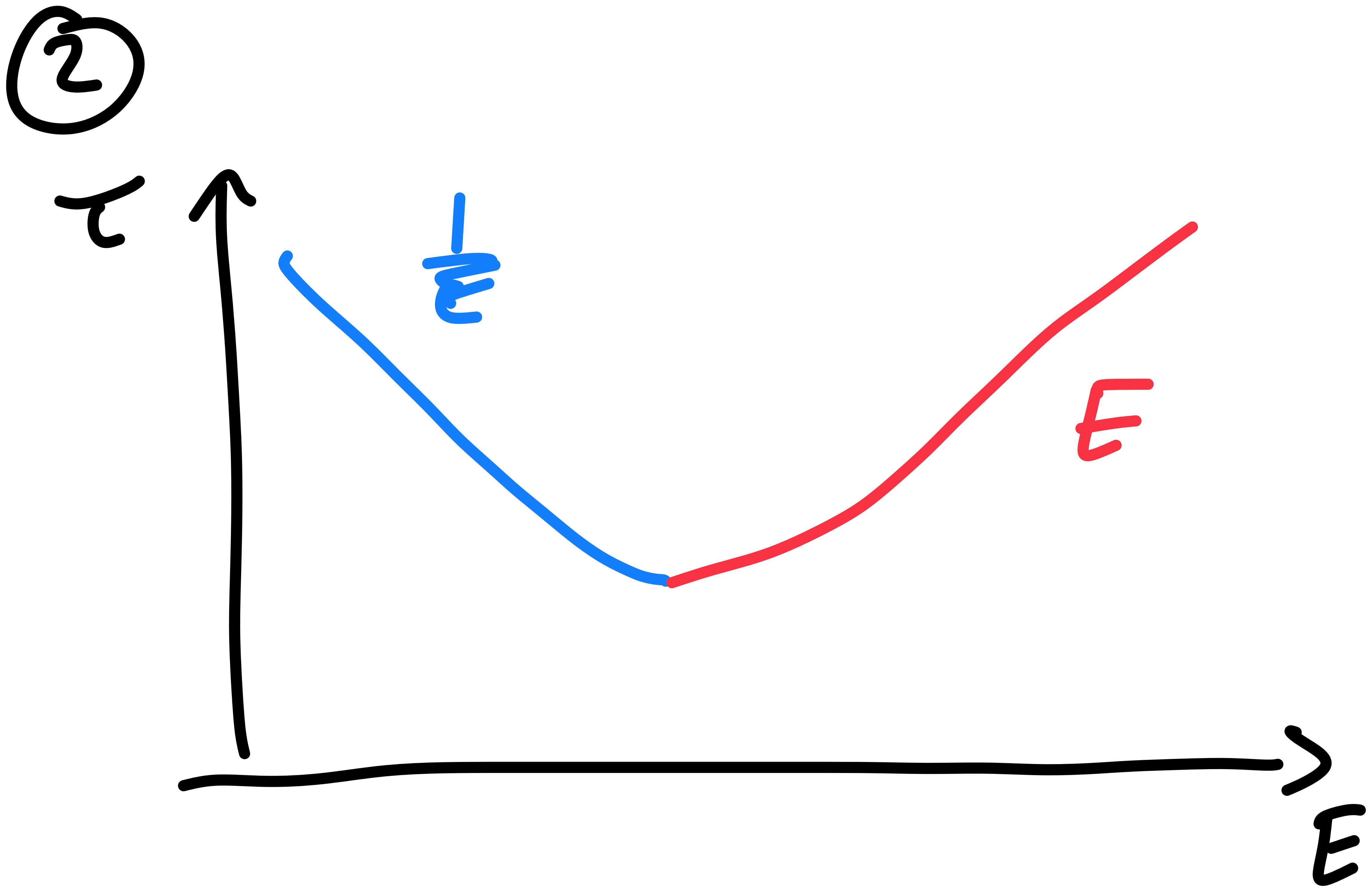
isotropic ↓ isotropic average

⇒ more complicated E-distr. can be built from this

Goals

for δ in E_{el}, $E_{\gamma in}$





We need:

① $\langle E_{\gamma\,out} \rangle = f(b)\, \frac{b}{1+b}\, E_{el}$

$b = 4\, \frac{E_{el}\, E_\gamma}{(mc^2)^2}$

② $\langle \sigma \rangle = \frac{\sigma_T}{1+b}$

Compton scattering

① electron at rest

② energy + momentum conservation

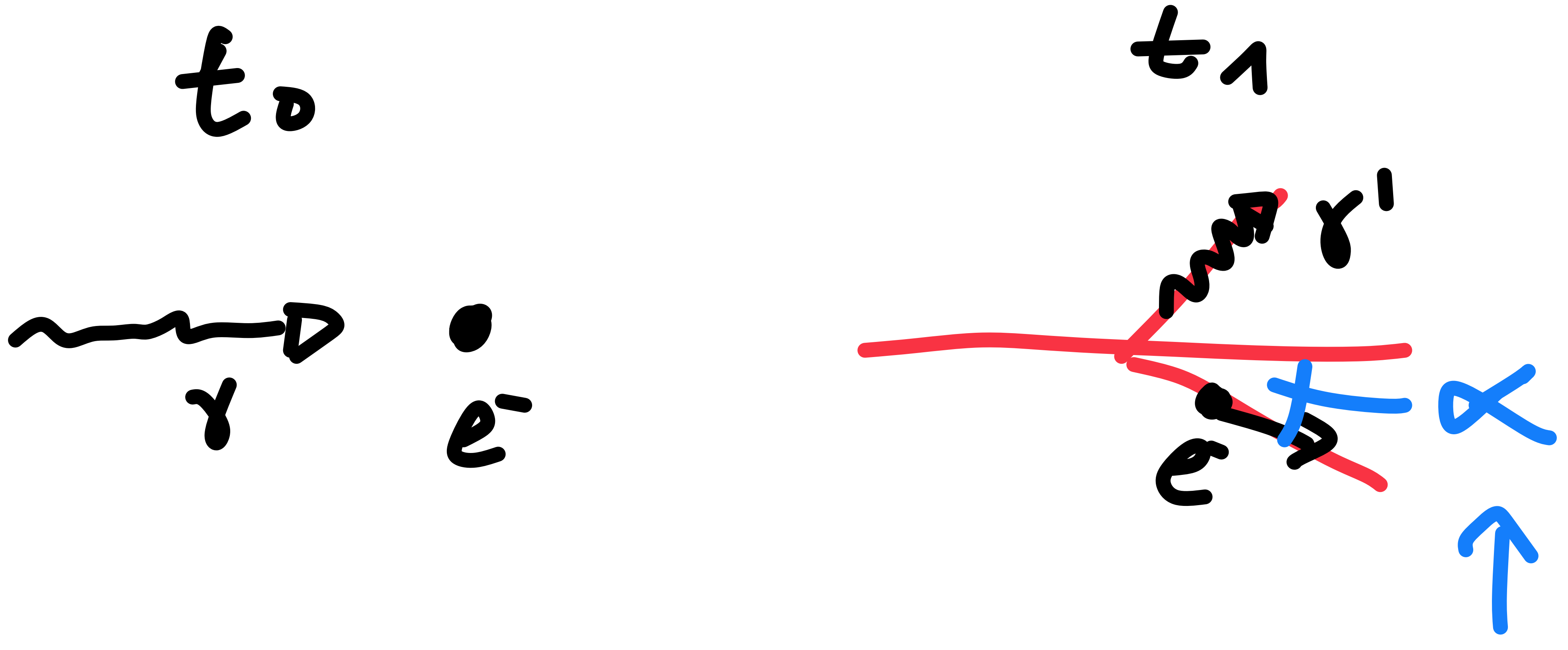
$$\Rightarrow \boxed{E_\gamma' = \frac{E_\gamma}{1 + \frac{E_\gamma}{mc^2}(1 - \cos\alpha)}}$$

$$\Downarrow$$

$$\frac{E_\gamma}{1 + 2\frac{E_\gamma}{mc^2}} < E_\gamma' < E_\gamma$$

back scattering

"no" scattering



scattering angle

Relativistic electron

→ ERF + lab frame

↓ ↓

E_{el}, E_γ

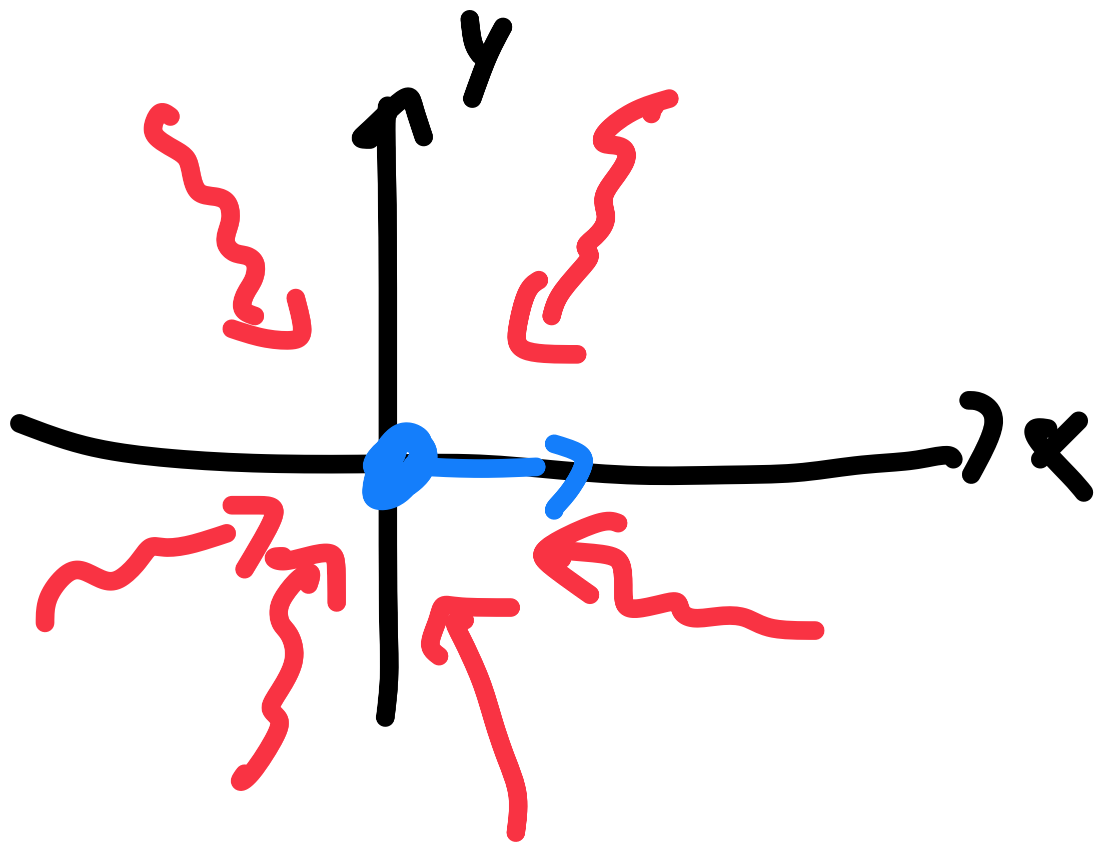
E'_γ ←

$$p_{el} = \begin{pmatrix} mc^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} \gamma mc \\ -\beta\gamma mc \\ 0 \\ 0 \end{pmatrix}$$

$$p_\gamma = E_\gamma \begin{pmatrix} 1/c \\ \cos\theta/c \\ \sin\theta/c \\ 0 \end{pmatrix}$$

1. boost to ERF → Γ
2. scatter → change direction
3. boost back to lab → another Γ

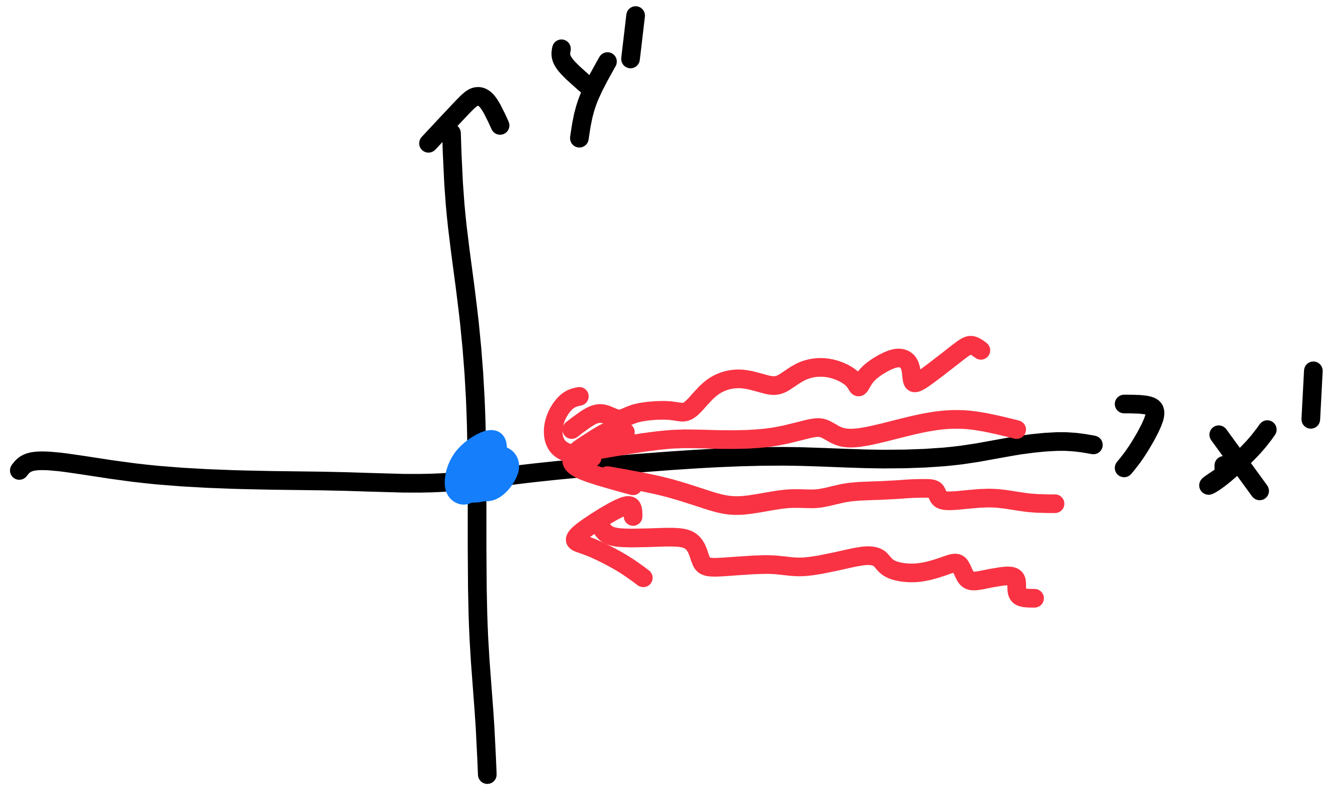
lab



boost in +x

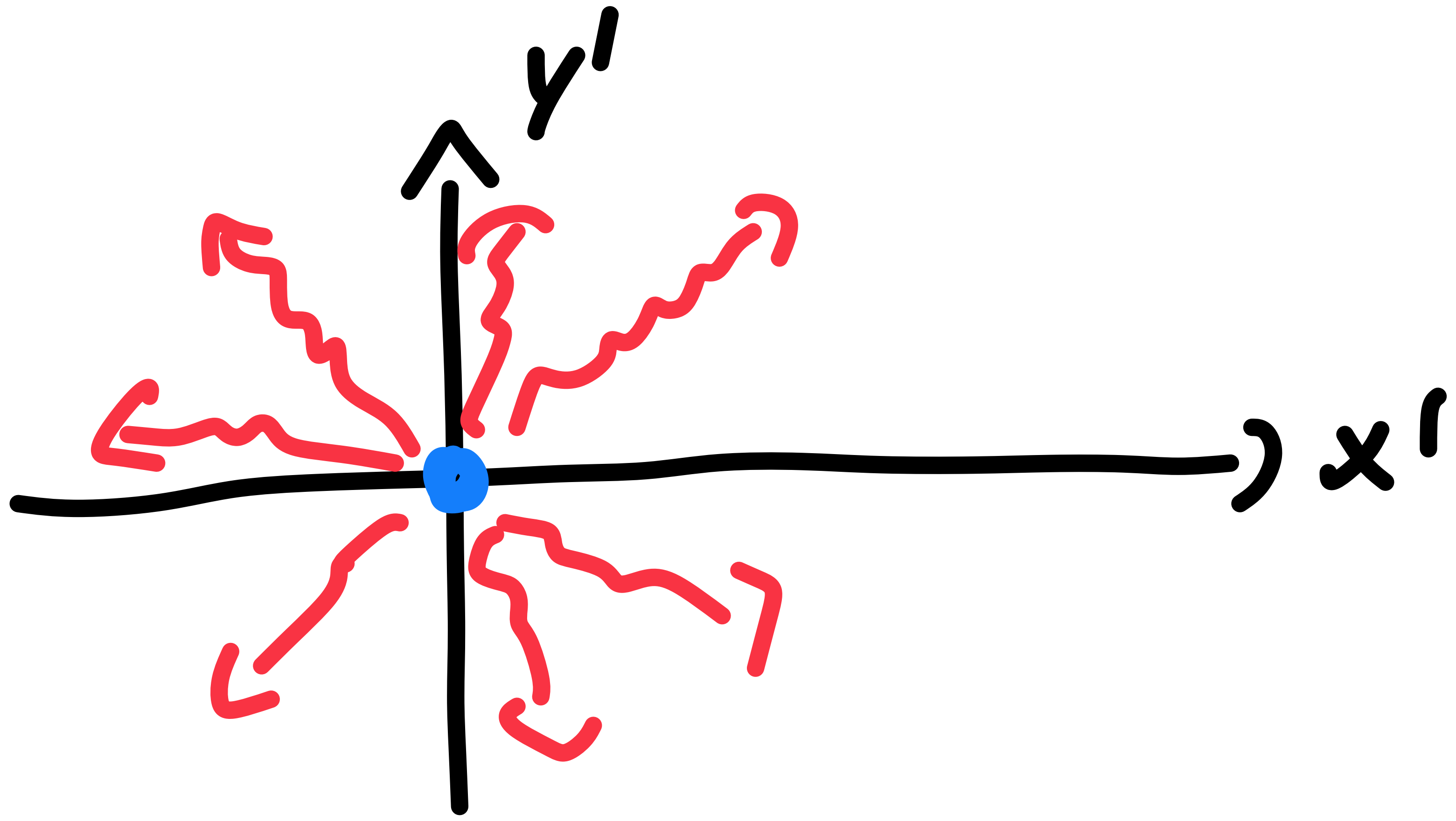
$$\Lambda = \begin{pmatrix} \Gamma & -\beta\Gamma & 0 \\ -\beta\Gamma & \Gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ERF



scatter

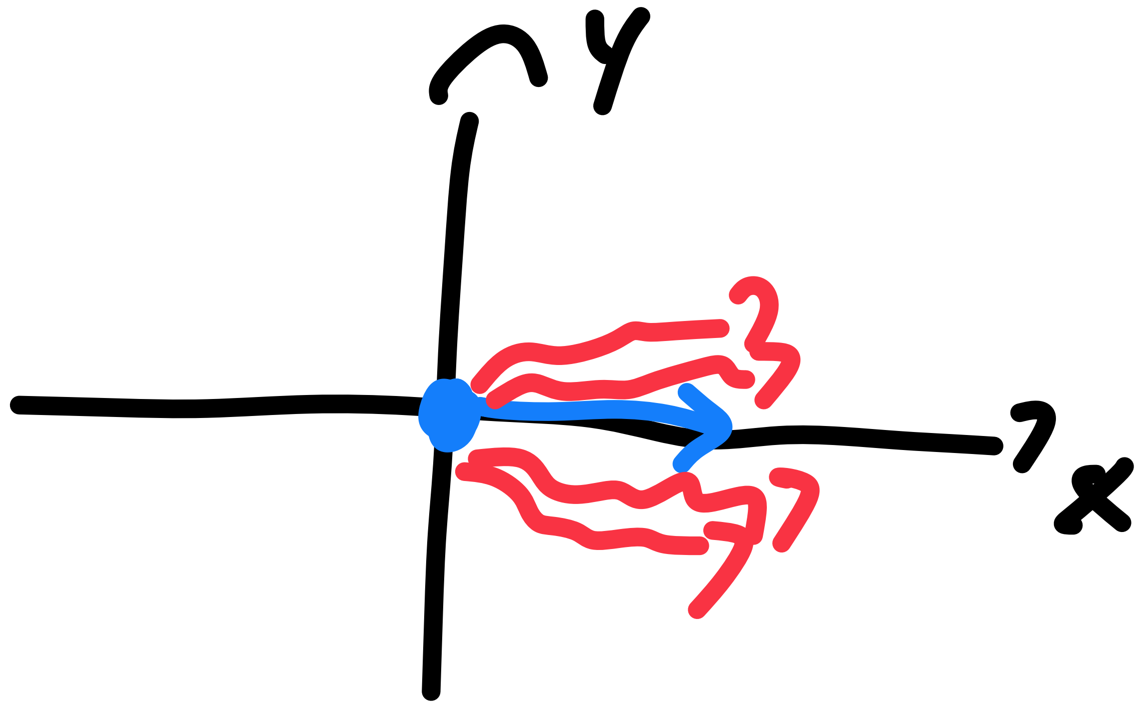
$$\frac{d\sigma}{dE'_\gamma \, d\cos\theta'}$$



boost in − x

$$\Lambda = \begin{pmatrix} \Gamma & \beta\Gamma & 0 \\ \beta\Gamma & \Gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

lab



Relativistic transformations

① change of energy $E_\gamma' = \Gamma(1-\beta\cos\theta)\, E_\gamma = \mathcal{D}\, E_\gamma$

② beaming (abberation) $\Rightarrow$ $\sin\theta' = \dfrac{\sin\theta}{\Gamma(1-\beta\cos\theta)} = \dfrac{\sin\theta}{\mathcal{D}}$

ultra-rel: $\Gamma \gg 1$

① increase in energy $E'/E = \Gamma(1-\beta\cos\theta)$

② $\dfrac{\sin\theta'}{\sin\theta} = \dfrac{1}{\Gamma(1-\beta\cos\theta)}$

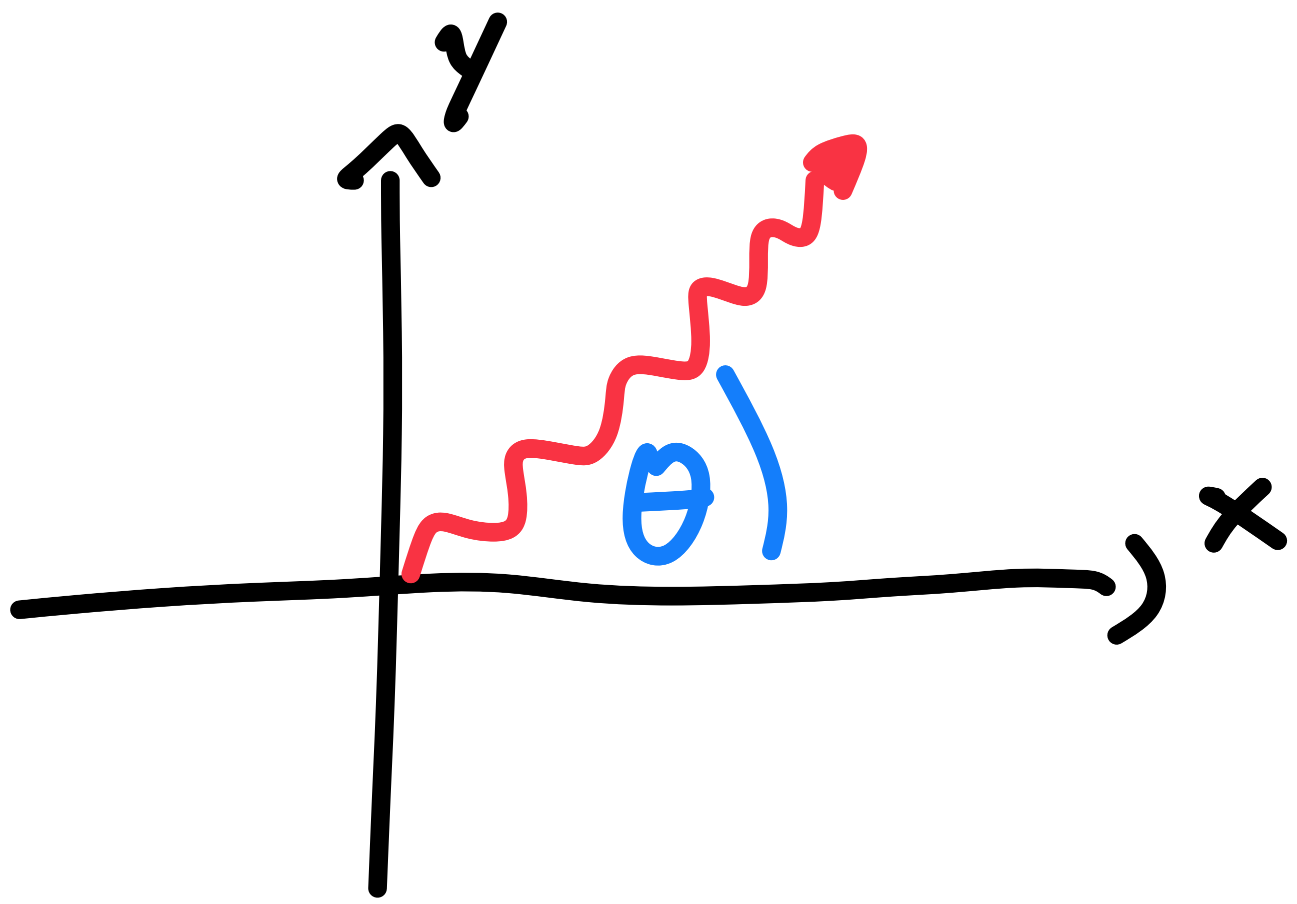
remember

source: E_γ, θ

obs: E_γ', θ'

$\mathcal{D} = \dfrac{1}{\Gamma(1-\beta\cos\theta')}$

$= \Gamma(1-\beta\cos\theta)$



$$E_\gamma = p_\gamma c$$

$$p_x = \frac{E_\gamma}{c}\cos\theta \qquad \Rightarrow \qquad P = \begin{pmatrix} E_\gamma \\ \frac{E_\gamma}{c}\cos\theta \\ \frac{E_\gamma}{c}\sin\theta \\ 0 \end{pmatrix}$$

$$p_y = \frac{E_\gamma}{c}\sin\theta$$

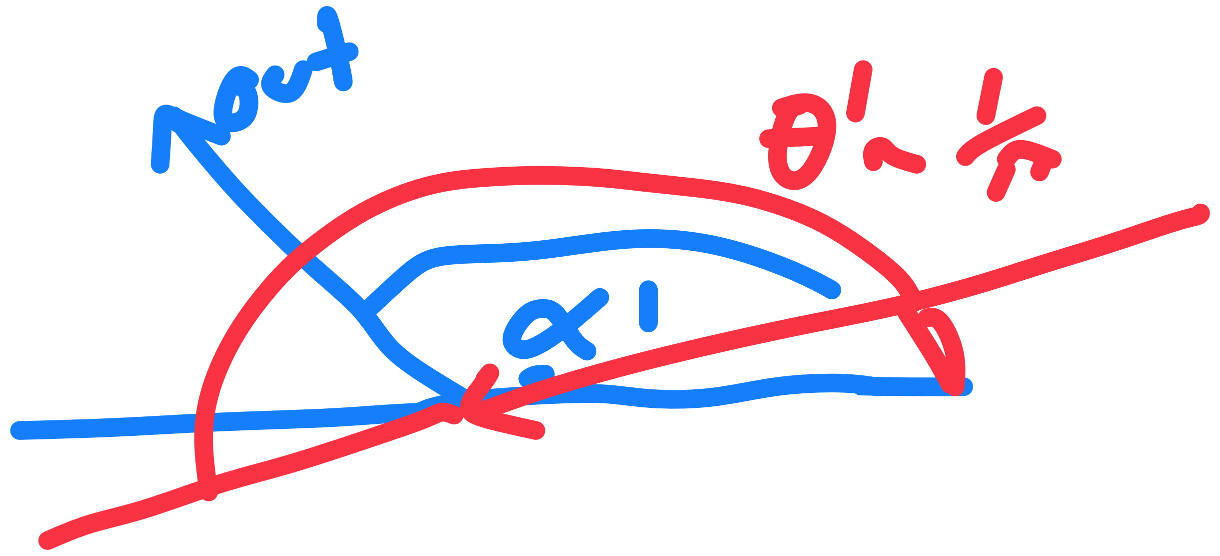
$\theta = 0$: parallel boost $\rightarrow \cos\theta = 1 \Rightarrow E'_\gamma / E_\gamma = \Gamma(1-\beta) \approx \frac{1}{2\Gamma} + \mathcal{O}(\ldots)$

$\theta = 180°$: antiparallel boost $\rightarrow \cos\theta = -1 \Rightarrow E'_\gamma / E_\gamma = \Gamma(1+\beta) \approx 2\Gamma$

inverse Compton scattering

$$E'_{\gamma,out} = \frac{E'_{\gamma in}}{1 + \frac{E'_{\gamma in}}{mc^2}(1-\cos\alpha')}$$

$$E'_{\gamma in} = E_{\gamma in}\,\Gamma(1-\beta\cos\theta)$$



$\phi' = \alpha' + \theta' - 180$

$$\Rightarrow E_{\gamma,out} = E'_{\gamma out}\,\Gamma(1+\beta\cos\phi')$$

$$= E_{\gamma in}\frac{\Gamma^2(1-\beta\cos\theta)(1+\beta\cos\phi')}{1+\frac{E_{\gamma in}}{mc^2}\Gamma(1-\beta\cos\theta)(1-\cos\alpha)}$$

outgoing photon energy: rewritten

$$E_{\gamma out} = \frac{\frac{4 \overbrace{\frac{E_{\gamma in} E_{el}}{(m_e c^2)^2}}^{b \to \frac{E_\gamma'}{m c^2} \text{ in ERF}} (1-\beta\cos\theta)(1+\beta\cos\phi')}{4} E_{el}}{1 + 4 \frac{E_{\gamma in} E_{el}}{(m_e c^2)^2} \frac{(1-\beta\cos\theta)(1-\cos\alpha')}{4}}$$

average
- θ iso.
- α' KN cross section

$$\approx f(b) \underbrace{\frac{b E_{el}}{1+b}}_{E_{\gamma out, max}}$$

$\underline{\text{min?}}$: "no scattering" in ERF (tail on)

→ boosts cancel out

→ $E_{\gamma out\, min} = E_{\gamma in}$

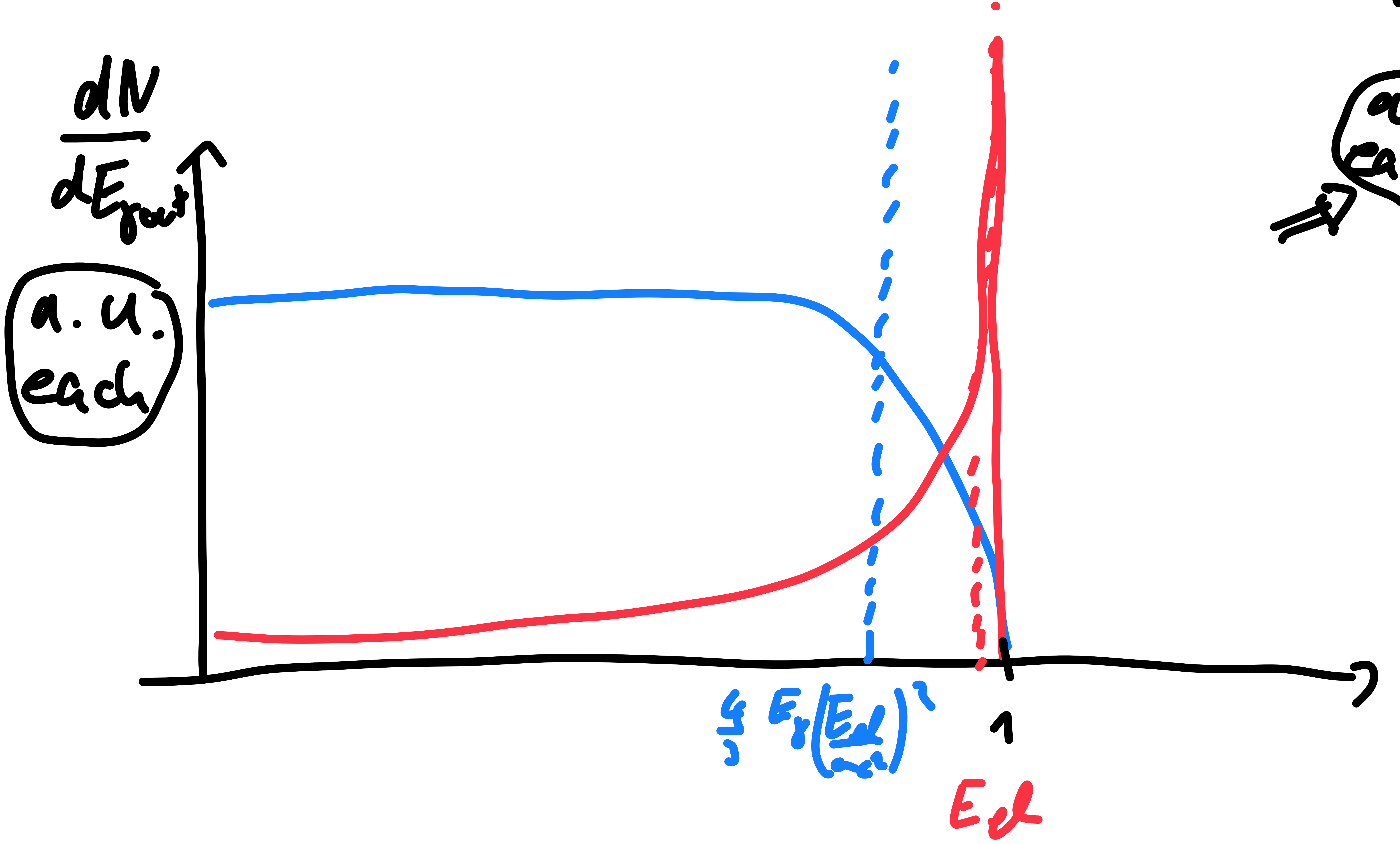
maximum Energy

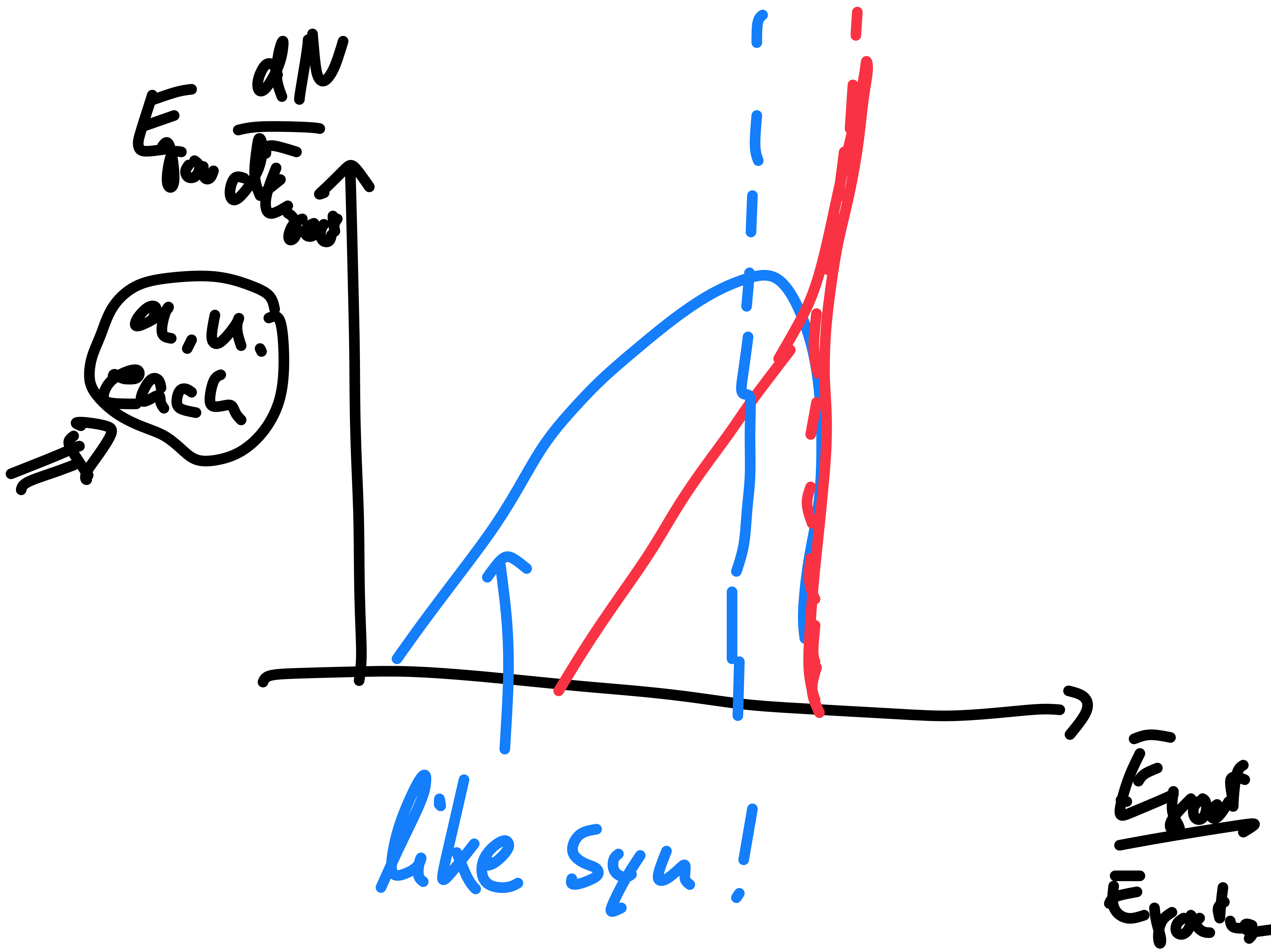
$$E_{\gamma out\,max} = \frac{b}{1+b} E_{el}$$

Thomson: $\boxed{b \ll 1} \Rightarrow E_{\gamma out\,max} = 4 E_\gamma \left(\frac{E_{el}}{m_e c^2}\right)^2$

KN: $\boxed{b \gg 1} \Rightarrow E_{\gamma out\,max} = E_{el} \rightarrow$ energy conservation

distribution (→ kernel F_{IC})





→ normalisation ?

total crosssection in lab frame :

in ERF : $\frac{d\sigma_{KN}}{d\Omega' dE'} = \frac{1}{2}\frac{\alpha^2}{m^2}\left(\frac{E'_{out}}{E'_{in}}\right)^2\left(\frac{E'_{out}}{E'_{in}} + \frac{E'_{in}}{E'_{out}} - \sin^2\alpha'\right)\delta\left(E'_{out} - \frac{E'_{in}}{1+\frac{E'_{in}}{mc^2}(1-\cos\alpha)}\right)$

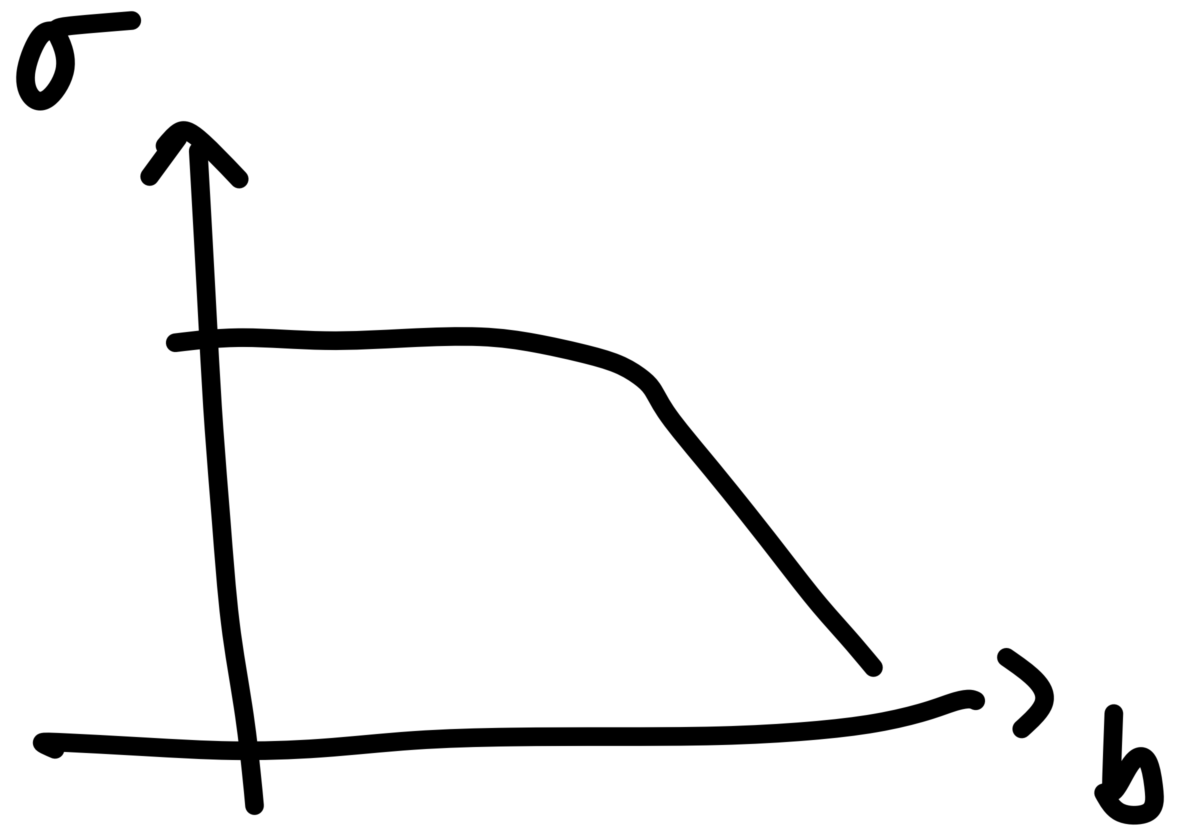
$(1-\cos\alpha')$

→ boost to lab frame $\frac{d\sigma_{KN}}{d\Omega\, dE}$

→ average again in α'

→ integrate in energy : $\sigma \approx \sigma_T \frac{\log(1+b)}{b} \approx \frac{\sigma_T}{1+b}$

→ same as integration of kernel F_{IC}



Conclusion

→ good qualitative description using

$$\boxed{\langle E_{\gamma out} \rangle = f(b)\, \frac{b}{1+b} E_{el}} \qquad \boxed{\sigma = \frac{\sigma_T}{1+b}}$$

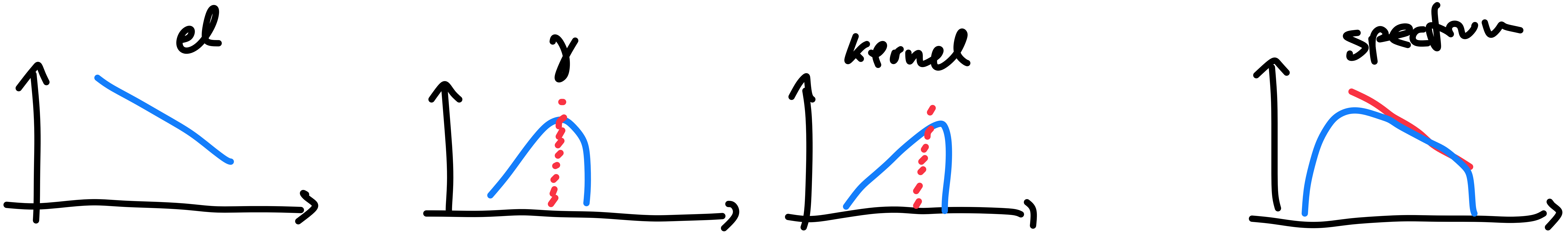
→ kernel: $\dfrac{dN}{dE_{\gamma out}} \approx \dfrac{c\,\sigma(b)}{V}\, \delta\left(E_{\gamma out} - \langle E_{\gamma out} \rangle\right)$

$$= \frac{c\sigma_T}{V} \frac{1}{1+b}\, \delta\left(E_{\gamma out} - f(b) \frac{b}{1+b} E_{el}\right)$$

arbitrary spectra:

consider (isotropic) distributions: $\frac{dN}{dE_{el}}$, $\frac{dN}{dE_\gamma}$

$$\Rightarrow \frac{dN}{dE_{\gamma out}} = \int dE_{el} \frac{dN}{dE_{el}} \cdot \int dE_\gamma \frac{dN}{dE_\gamma} \cdot \frac{dN}{dE_{\gamma out}}$$



Convolutions:

consider $E\frac{dN}{dE} = \frac{dN}{d\ln E}$

approx

$$\Rightarrow \int dE\, \frac{dN}{dE}\, K = \int d\ln E\, \frac{dN}{d\ln E}\, K \approx \sum \underbrace{\delta\ln E}_{\sigma(1)} \frac{dN}{d\ln E}\, K$$

$$\approx \left(\frac{dN}{d\ln E} \cdot K\right)\Big|_{max}$$

δ

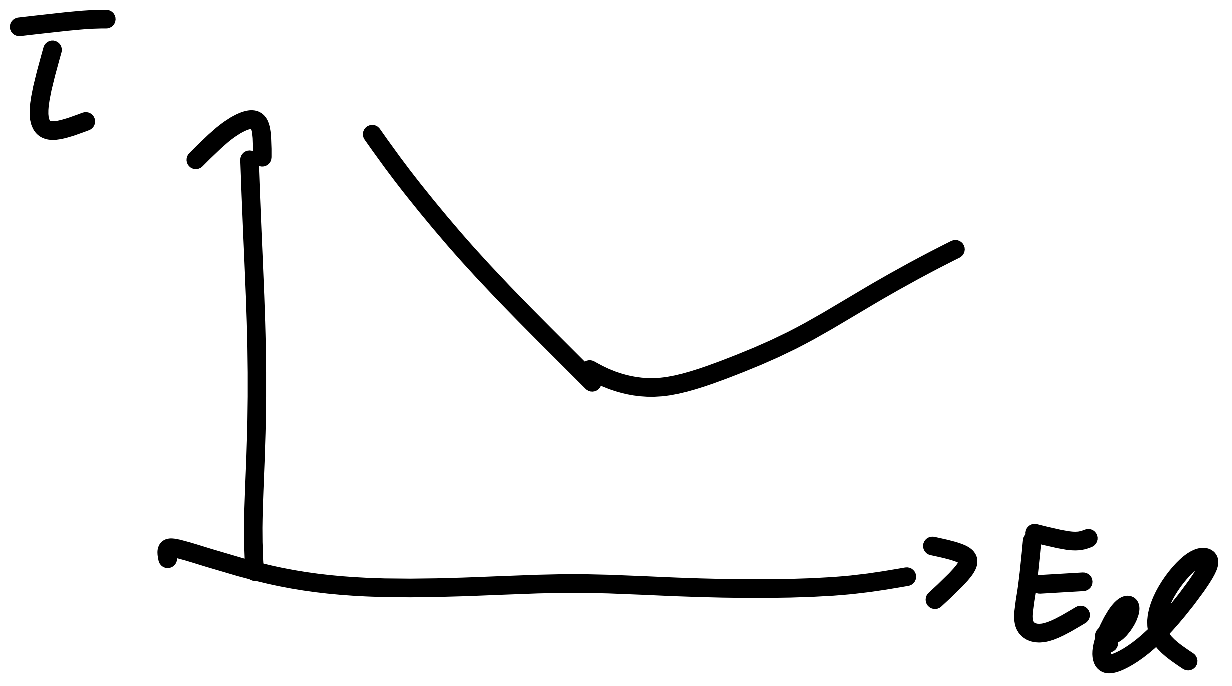
$$\rightarrow \int dE\, \frac{dN}{dE}\, \underset{\delta}{\underset{\downarrow}{K}} \approx$$

cooling time: (electron on single photon energy)

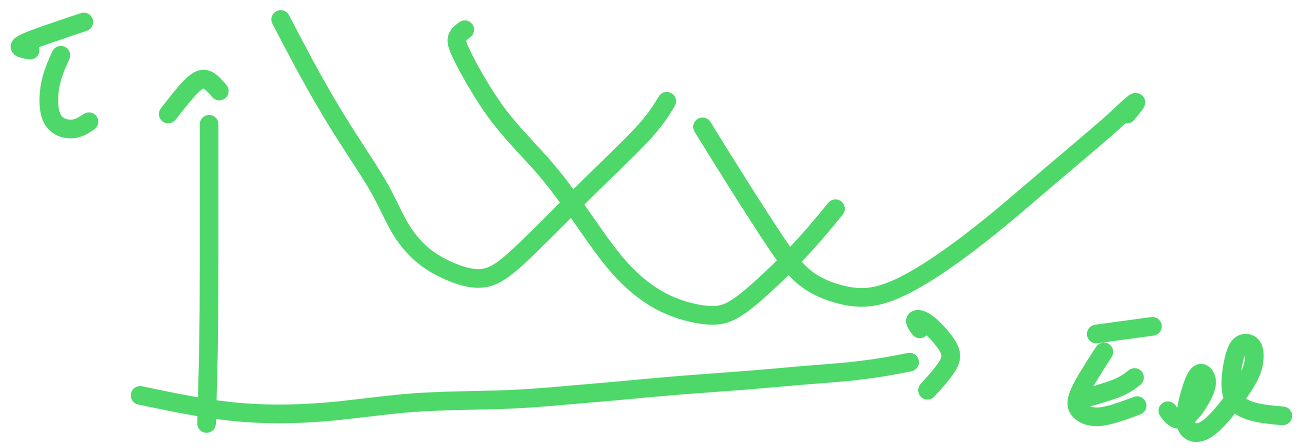
→ in general: $\tau = \frac{E}{\frac{dE}{dt}} = \frac{E}{R_{int} \cdot \Delta E} = \frac{1}{n \sigma \beta c \, \frac{\Delta E}{E}}$

$n\sigma \to \sim \frac{\sigma_T}{1+b}$, $\quad \beta \to 1$, $\quad \frac{\Delta E}{E} \to \sim \frac{b}{1+b}$

$$\Rightarrow \tau \approx \frac{(1+b)^2}{n \sigma_T c \, b} \quad \begin{cases} \xrightarrow{b \ll 1} \sim E^{-1} \\ \xrightarrow{b \gg 1} \sim E \end{cases}$$



for multiple peaks in E_γ ⇒ convolution



$$\tau^{-1} = \sum_i \tau_i^{-1}$$

for $\partial_E \left(\frac{E}{\tau} N \right)$