One Zone Basics and Effective Descriptions

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16.05.2022

VHE GRB Workshop 2022, Berlin

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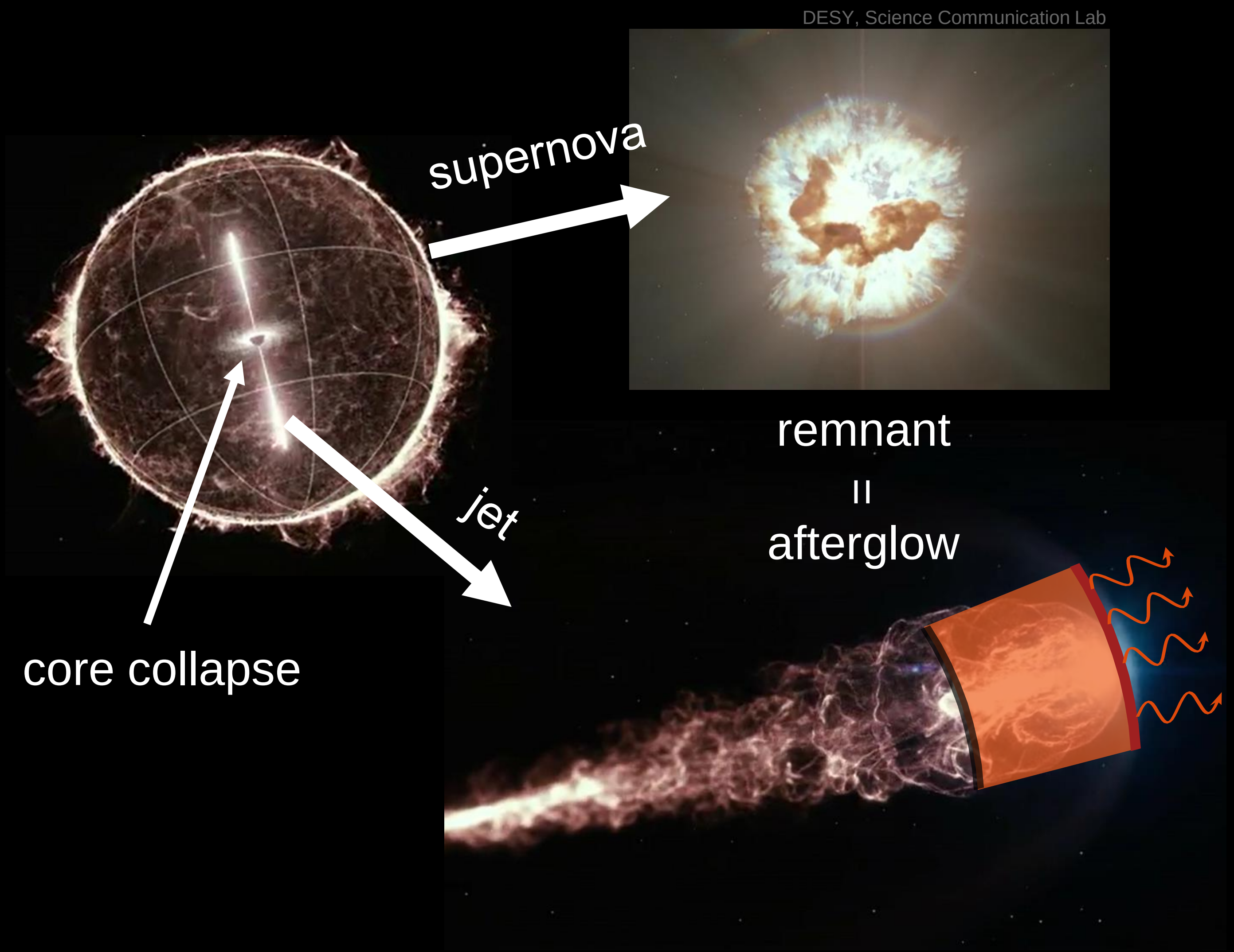
DESY.

Fireball model: Long GRB



- Lorentz factors up to few 100

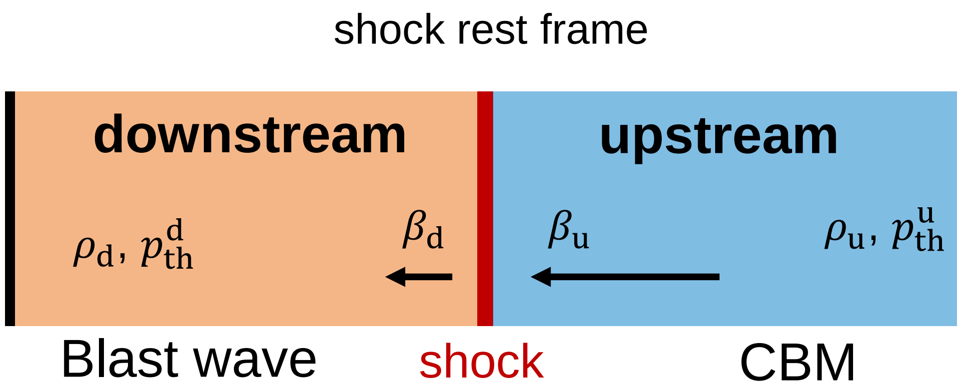
  → relativistic compression
- Quasi isotropic outflow
- Energetics:

  → observed up to: $E_{\mathrm{iso}} \sim 10^{54} erg$

  → $E_{\mathrm{tot}} = \frac{\Omega}{4\pi} E_{\mathrm{iso}} \sim 10^{51} \mathrm{erg}$

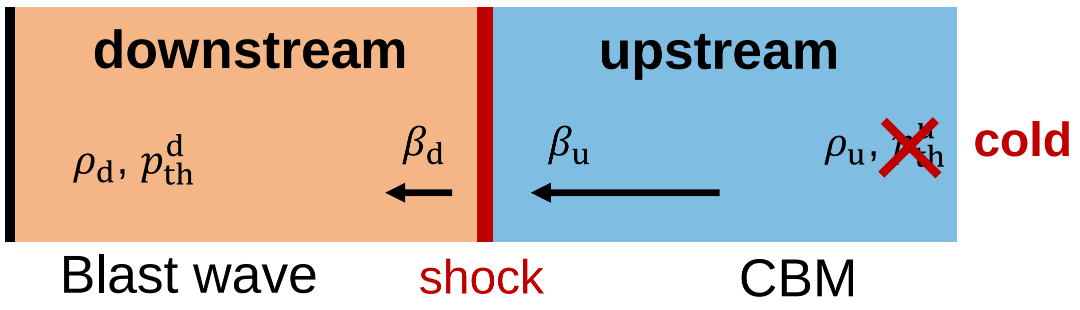
  → comparable to SN !
- efficient converters of kinetic energy to radiation

Main Progenitor: Shock



Shocks redistribute upstream ram pressure

shock rest frame

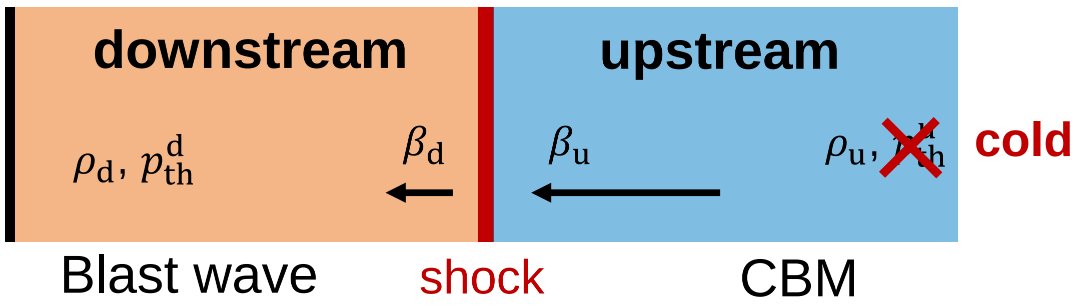


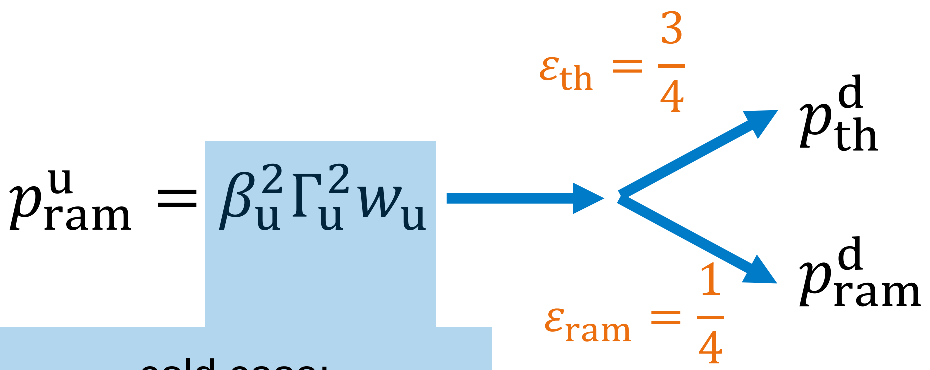
$$p_{ram}^u = \beta_u^2 \Gamma_u^2 w_u \rightarrow p_{th}^d, \; p_{ram}^d$$

cold case:
enthalpy density $w_u \approx \rho_u$

Non-relativistic shocks

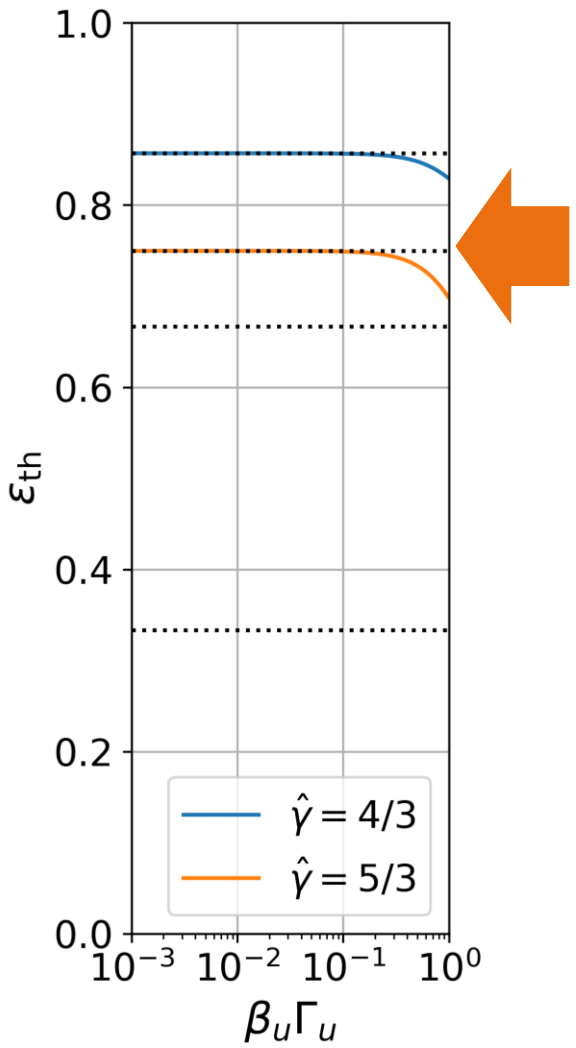
shock rest frame





cold case:
enthalpy density $w_u \approx \rho_u$

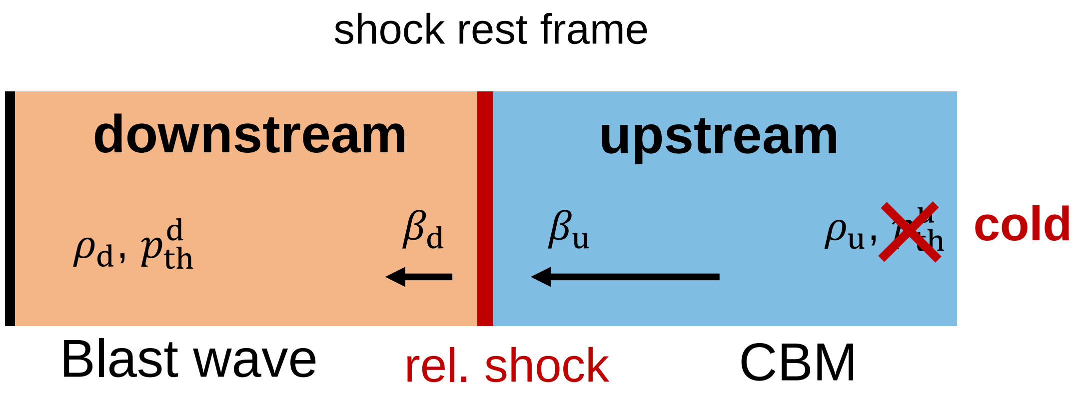
$$\varepsilon_X = \frac{p_X^d}{p_{ram}^u}$$



- non-rel. fluid ($\hat{\gamma} = 5/3$)
- non-rel. shock speed
- strong shock ($p_{th}^u / \rho_u \ll 1$)

$\rightarrow \varepsilon_{th} = 3/4$

Relativistic shocks

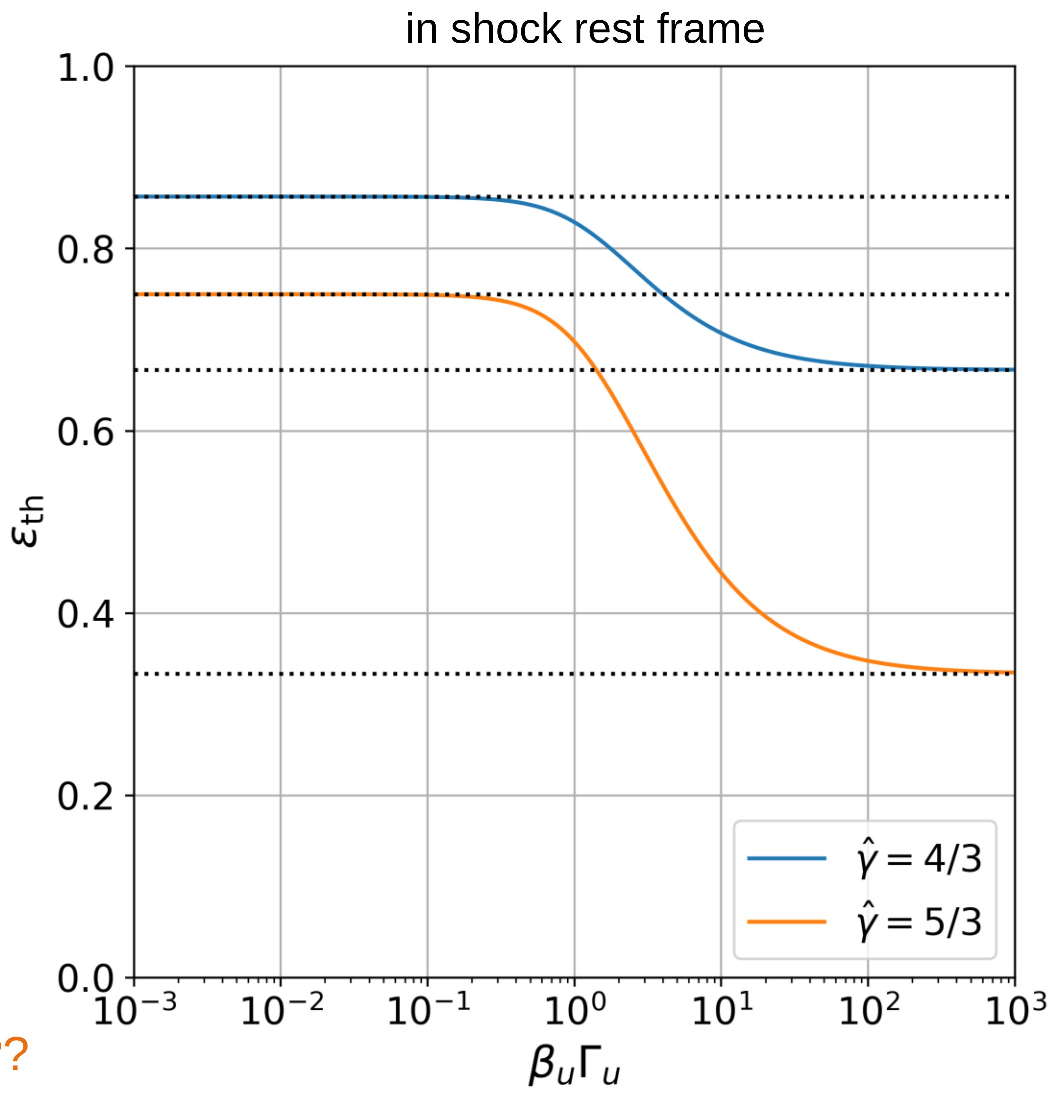


$p_{ram}^u \rightarrow$

- p_{th}^d $\quad \varepsilon_{th} = \frac{2}{3}$
- p_{ram}^d $\quad \varepsilon_{ram} = \frac{1}{3}$
- $p_{e,non-th}^d$ $\quad \varepsilon_e$ =few %?
- $p_{p,non-th}^d$ $\quad \varepsilon_p$ =few %?
- p_B^d $\quad \varepsilon_B = 10^{-4} - 10^{-2}$??

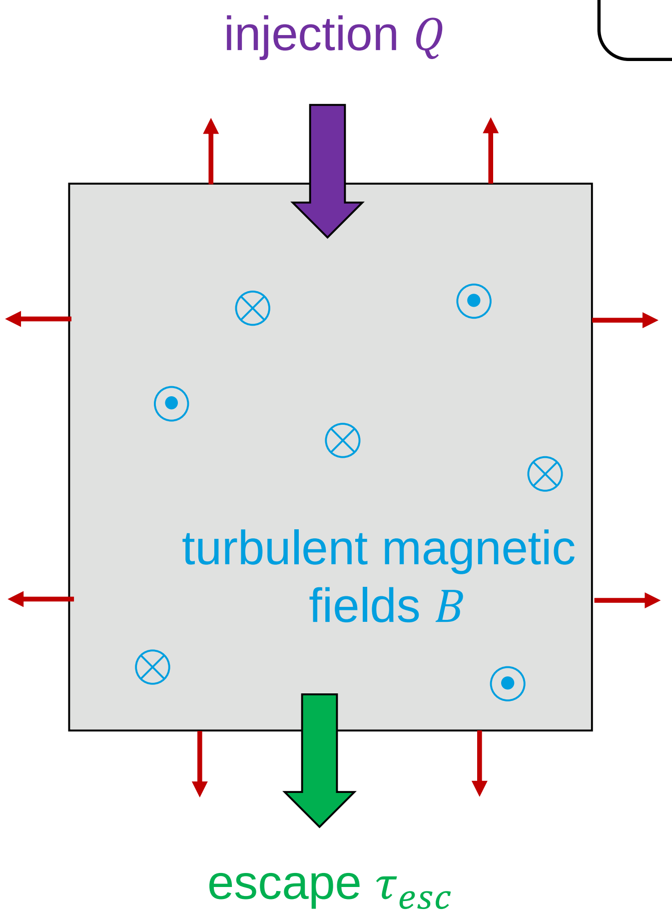
$$\varepsilon_X = \frac{p_X^d}{p_{ram}^u}$$

(can also define ε via energy density)



$$\frac{2}{3}$$

One Zone Modelling



electrons drive expansion of box

shock injects relativistic particles

electron number

$$\partial_t N_{E,el} = \partial_E \left[\left(\frac{E}{\tau_{syn}(E,t)} + \frac{E}{\tau_{iC}(E,t)} + \frac{E}{\tau_{adi}(t)}\right) N_{E,el}\right] + Q_{inj}(E,t) - \frac{N_{E,el}}{\tau_{pp}(E,t)} + Q_{pa}(E,t)$$

synchrotron

inverse Compton scattering

$e^- e^+ \rightarrow \gamma\gamma$

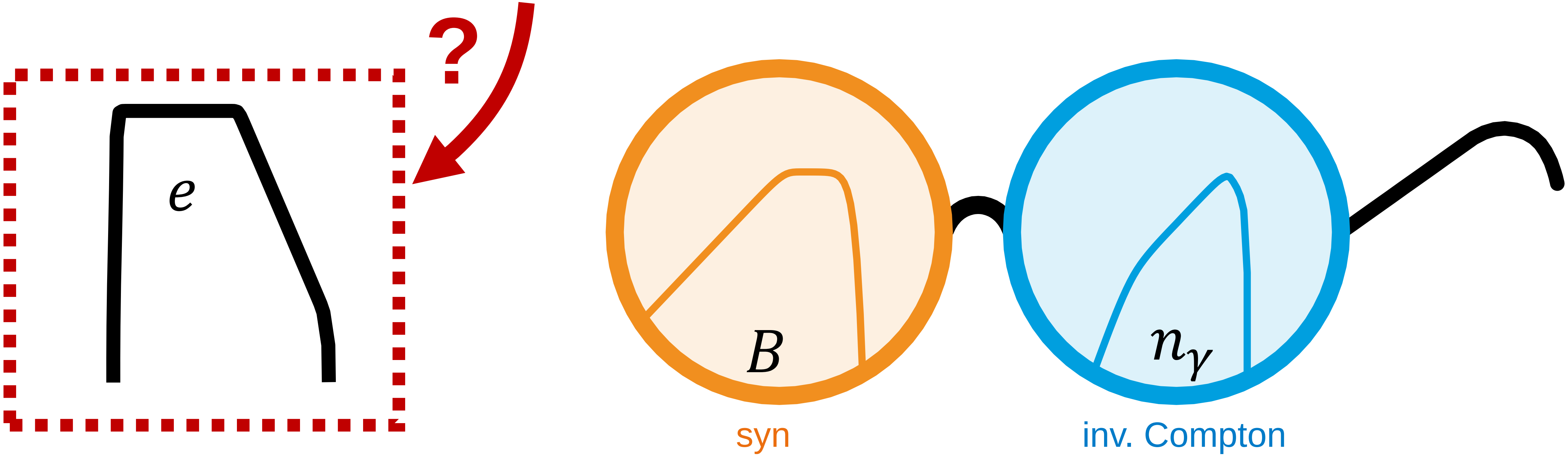
$\gamma\gamma \rightarrow e^- e^+$

photon number

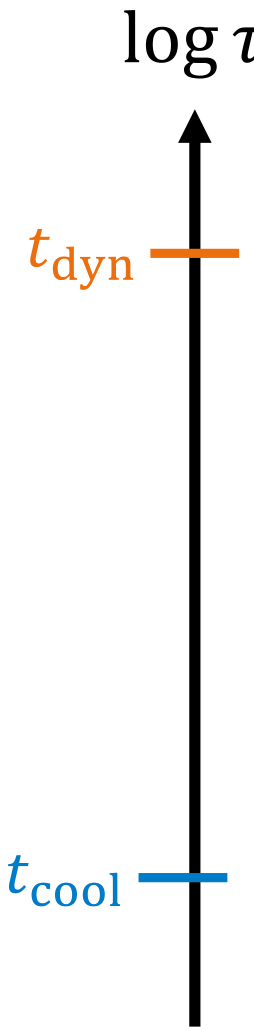
$$\partial_t N_{E,\gamma} = -\frac{N_{E,\gamma}}{\tau_{esc}(t)} + Q_{syn}(E,t) + Q_{iC}(E,t) + Q_{pp}(E,t) - \frac{N_{E,\gamma}}{\tau_{pa}(E,t)}$$

photons can escape from box

Is it motivated to model the **electron spectrum** as an effective **smoothly broken power law?**



Relevant time scales



t_{dyn}: time scale at which parameters of one zone change
(≈ same for B, V,…)

t_{cool}: time scale at which particles loose their energy
("lifetime in system")

Steady state case

∞ $\log \tau$

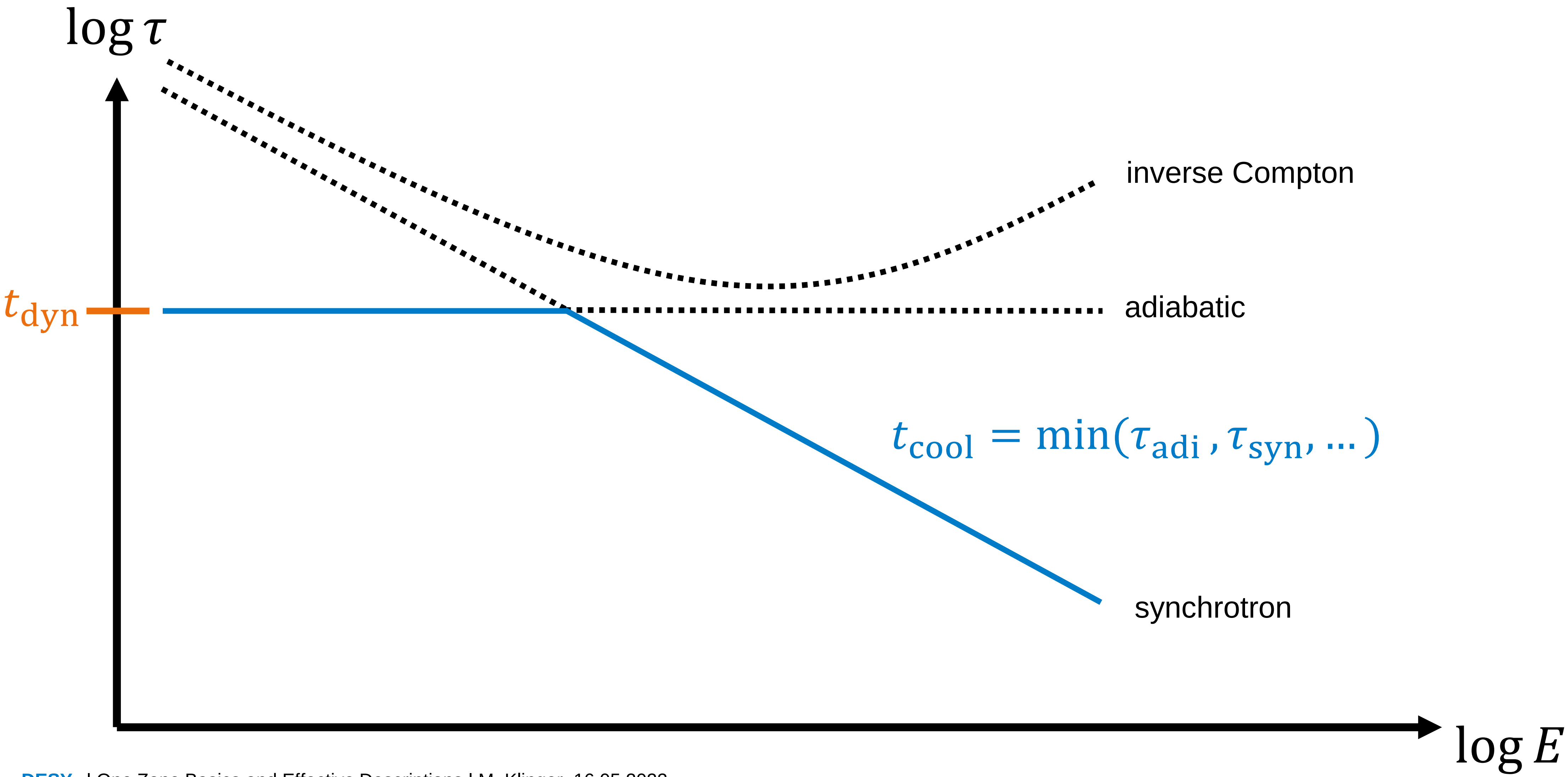
t_{dyn} time scale at which parameters of one zone change
($\approx$ same for B, V,…)

$t_{\mathrm{cool}} \ll t_{\mathrm{dyn}}$

steady state: parameters of the one zone are const
→ const steady state spectrum $N_E \sim Q_E \min(\tau_i)$

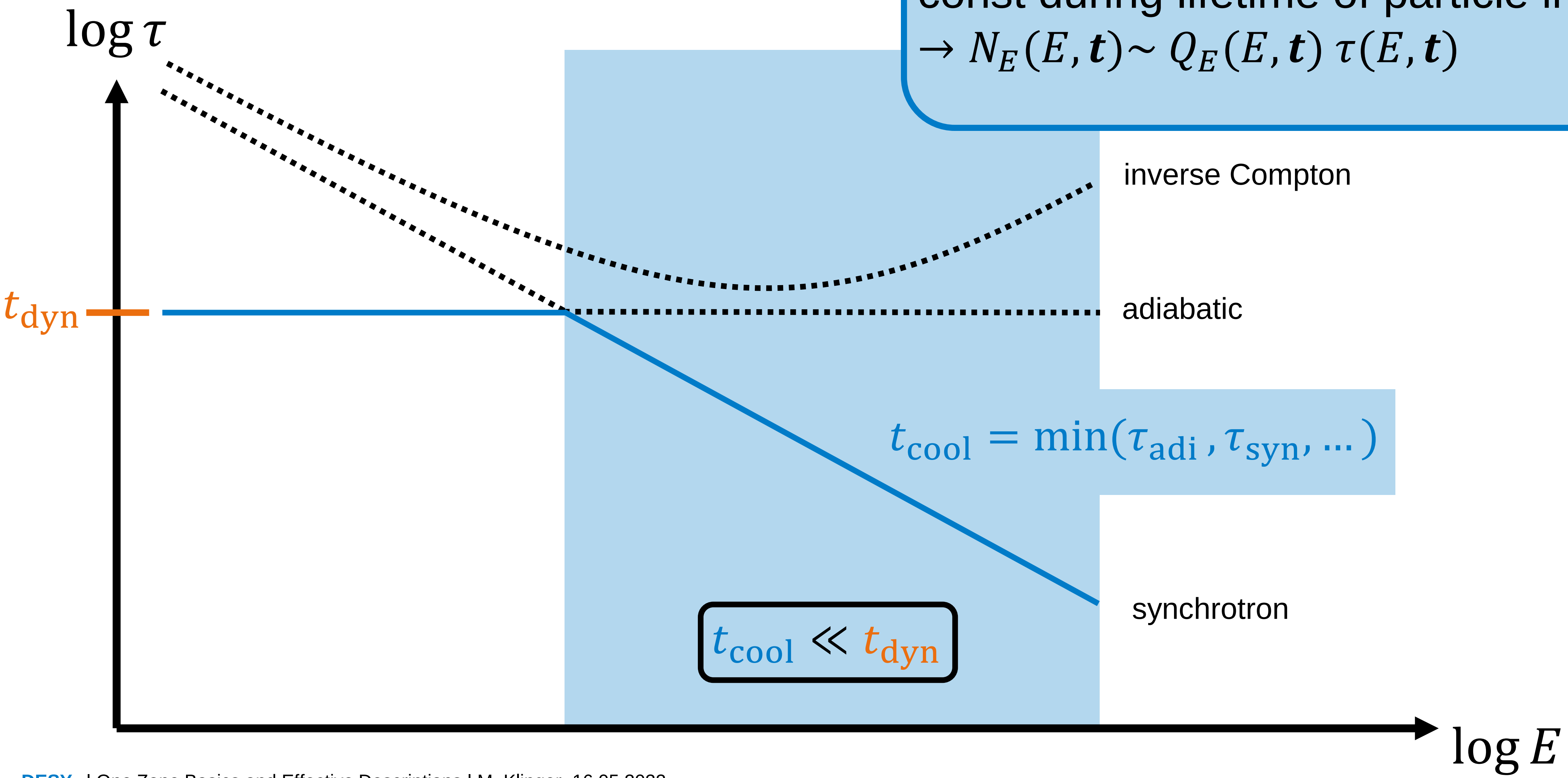
t_{cool} time scale at which particles loose their energy
(“lifetime in system”)

Time dependent modeling



Time dependent modeling

quasi-steady state:
parameters of the one zone are approx. const during lifetime of particle in system
$\rightarrow N_E(E,\boldsymbol{t}) \sim Q_E(E,\boldsymbol{t})\,\tau(E,\boldsymbol{t})$



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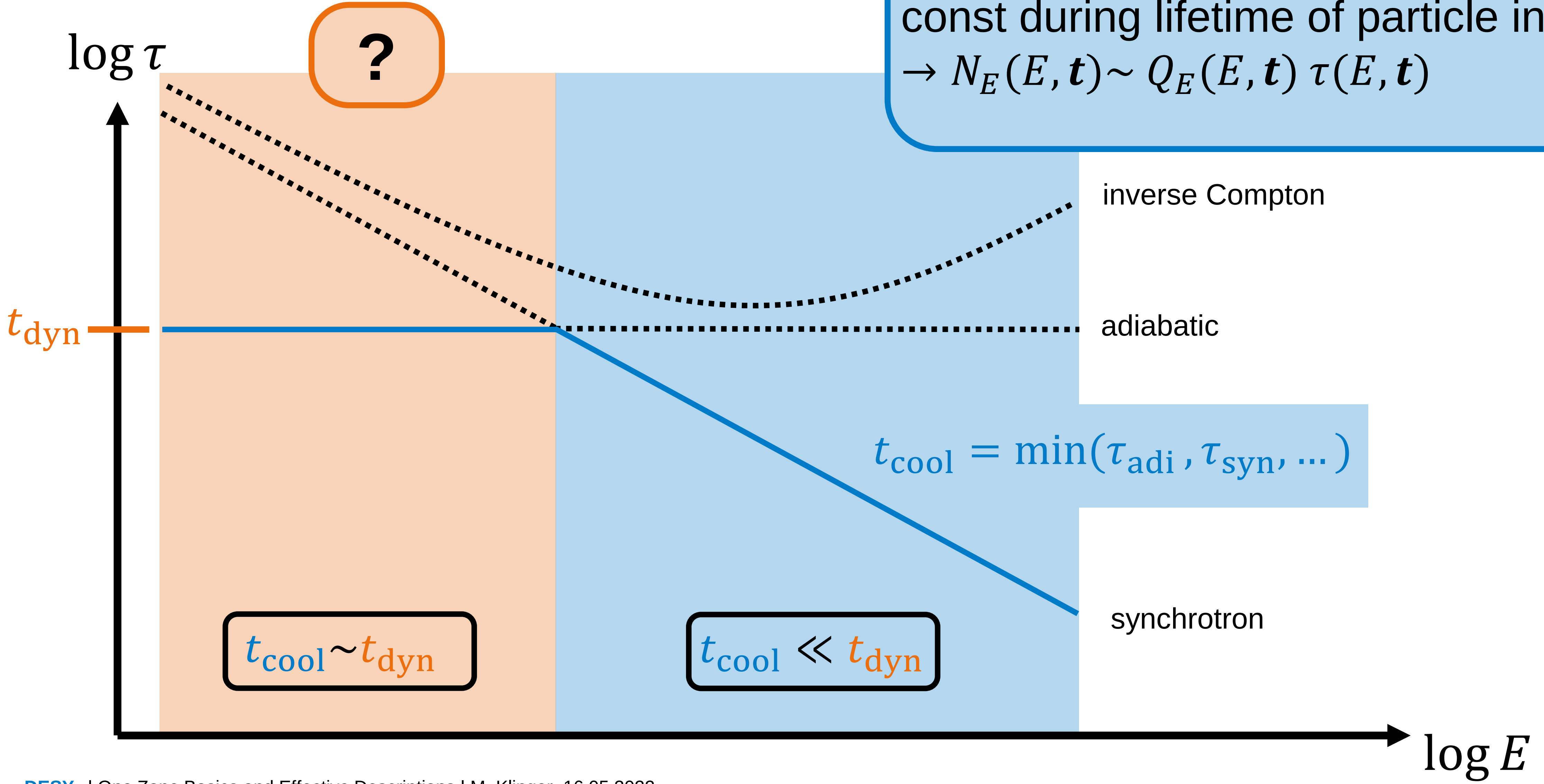

Adiabatic cooling regime ($\tau_{adi} \ll \tau_i$)

$$\longrightarrow \quad \partial_t N_{E,el} = \partial_E \left(\frac{E}{\tau_{\mathrm{adi}}(t)} N_{E,el} \right) + Q_E(E,t) \qquad \text{(PDE)}$$

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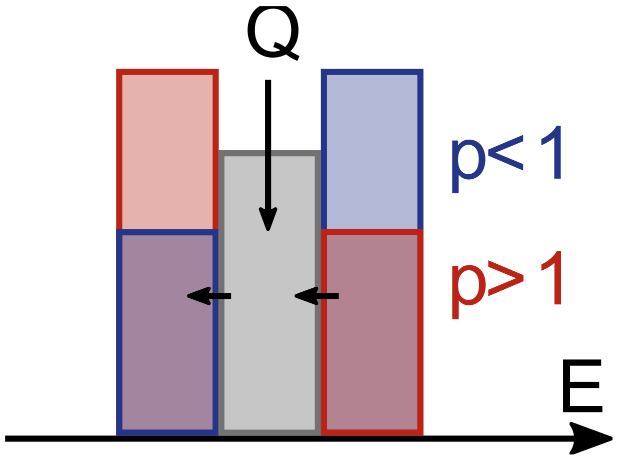
For intuition:
cooling term
≈ effective escape term

- $\tau_{adi}(t)$ only a function of time
- injection power law in E, $Q_E \sim E^{-p}$
- $N_E \sim E^{-p}$

$$\approx -(p-1)\frac{N_{E,el}}{\tau_{\text{adi}}(t)} := -\frac{N_{E,el}}{\tau_{\text{eff}}(t)}$$

$$\partial_t N_{E,el} + \frac{N_{E,el}}{\tau_{\text{eff}}(t)} = Q_E(E,t) \quad (\approx\text{ODE})$$



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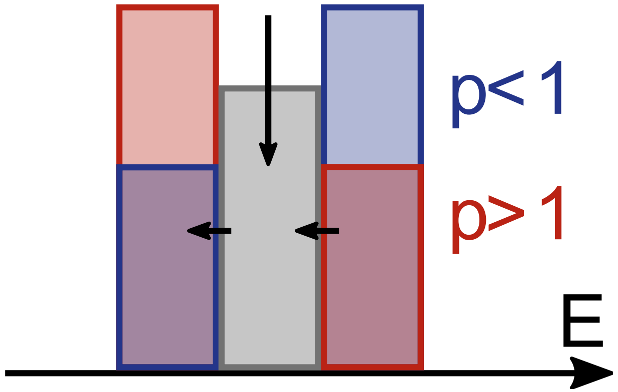

Green's function in time?

→ how does $\tau_{\text{eff}} = \frac{\tau_{\text{adi}}}{p-1}$ scale with time?

One Zone Parameter’s time dependence

- it’s all about the deceleration of the shock: $r(t_{\text{obs}}), \Gamma(t_{\text{obs}})$

1. conservation of energy: initial $E_0 = \Gamma^2 M_{\text{sw}}(r) c^2$ heated swept up material
2. assume density profile $n(r) \sim r^{-w} \rightarrow \Gamma(r) \sim r^{-\frac{3-w}{2}}$
3. from Doppler boosting $t_{\text{obs}} \approx \int \frac{\text{d}r}{2\beta c \Gamma^2} \rightarrow r \sim t_{\text{obs}}^{\frac{1}{4-w}}$

$\rightarrow r(t_{\text{obs}}), \Gamma(t_{\text{obs}}) \sim \left(\frac{E_0}{t_{\text{obs}}^{3-w}}\right)^{\frac{1}{2(4-w)}}$ $\rightarrow$ ISM ($w = 0$): $r \sim t_{\text{obs}}^{1/4}$, $\Gamma \sim \left(\frac{E_0}{t_{\text{obs}}^{3}}\right)^{\frac{1}{8}}$

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- adiabatic cooling from size $\Delta \sim \frac{r}{\Gamma}$ (in "comoving" frame)

  $\rightarrow \tau_{\text{adi}}(t_{\text{co}}) = \cdots = \frac{6}{9-w} \frac{5-w}{2} t_{\text{co}}$ **→ Green's function in time?**

  comoving time $t_{\text{co}} = \int \frac{\mathrm{d}r}{\beta c \Gamma} \sim \frac{r}{\Gamma c}$

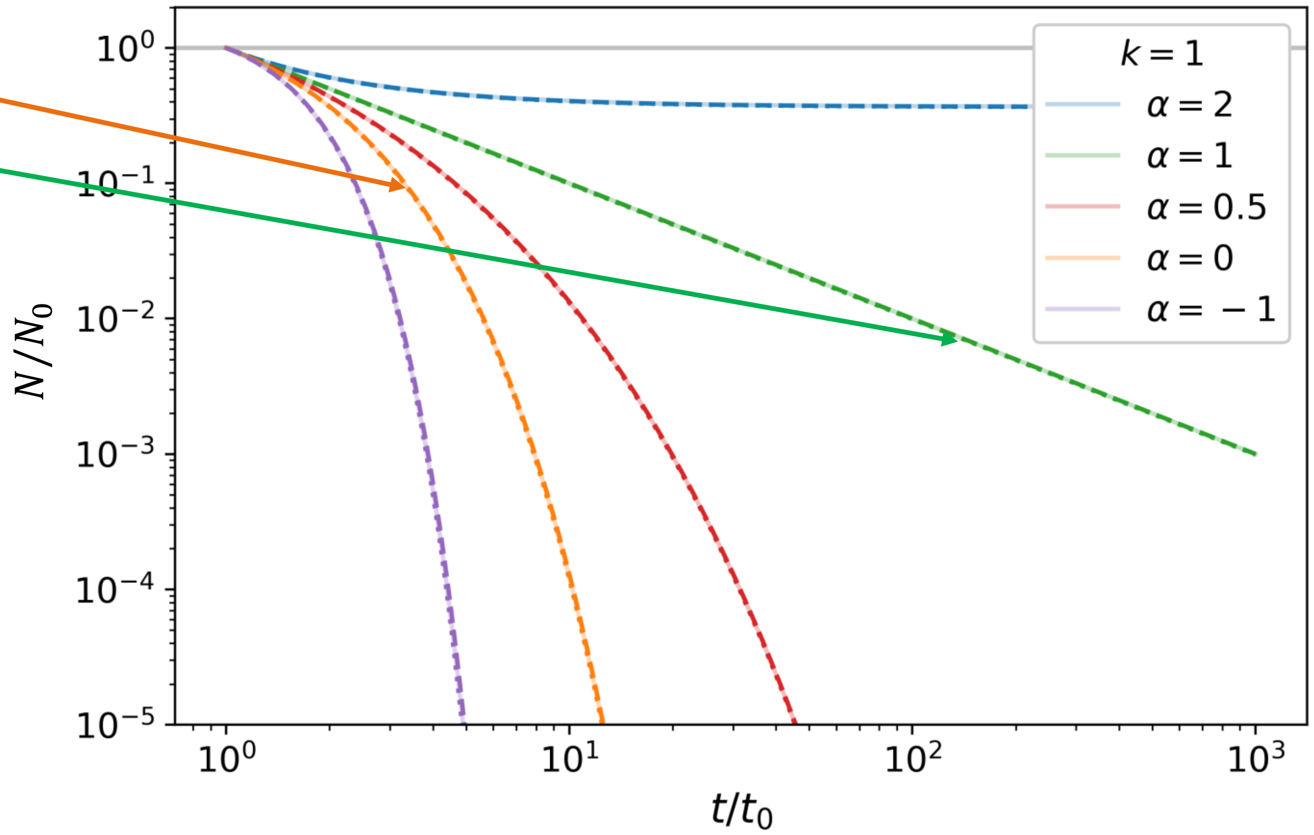
Green's function for power law τ

- $$\partial_t N_E = -\frac{N_E}{\tau_0\left(\frac{t}{\tau_0}\right)^{\alpha_\tau}}$$

  → for $\alpha_\tau = 0$: exponential decay

  → for $\alpha_\tau = 1$: power law decay

- relevant to see if early/late injections dominate spectrum



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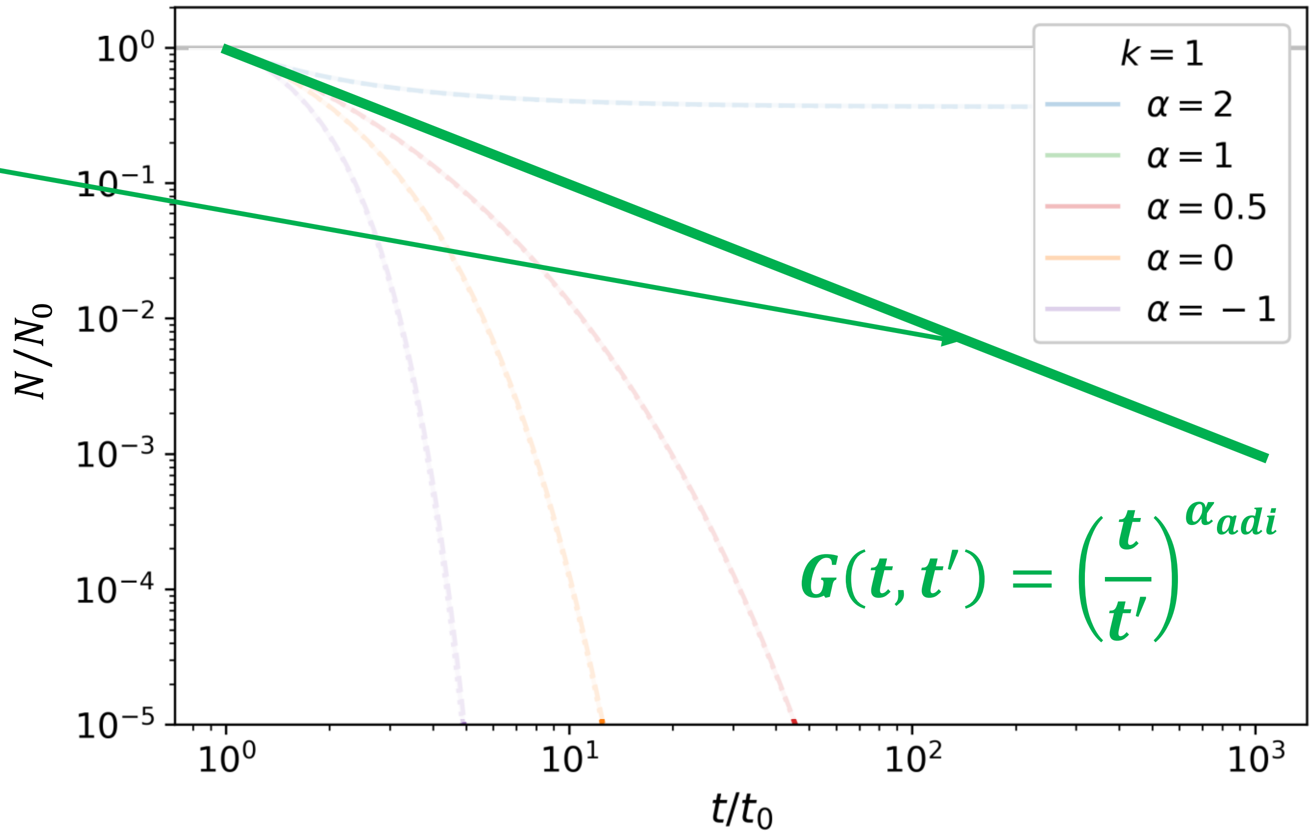


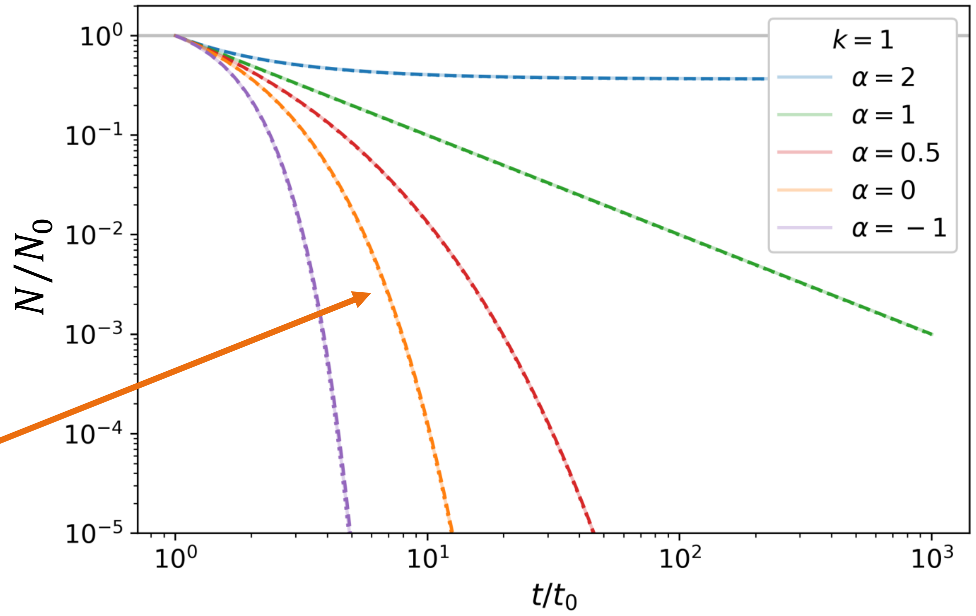
caution: $\alpha_G \neq \alpha_\tau$

- α_τ → shape
- α_{adi} → slope for $\alpha_\tau = 1$

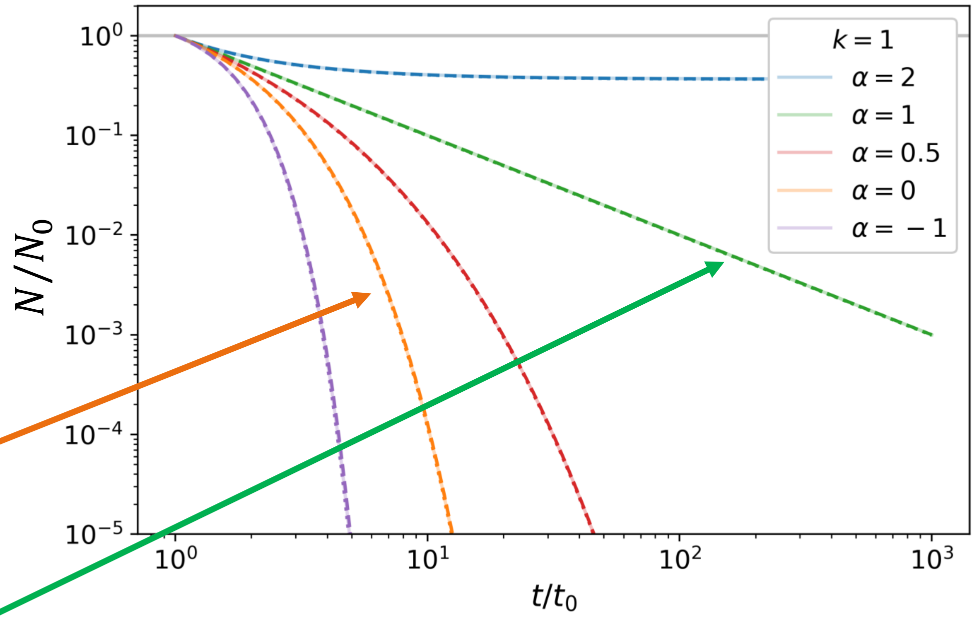
(Quasi) steady state

- recap: **steady state**:
  → parameters constant ($\alpha_\tau = 0$)

$\partial_t N_E = -\frac{N_E}{\tau} + Q_E \quad \rightarrow \quad N_E = Q_E \tau \cdot (1 - e^{-t/\tau})$



(Quasi) steady state



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$$\partial_t N_E = -\frac{N_E}{\tau} + Q_E \qquad \rightarrow \qquad N_E = Q_E \tau \cdot \left(1 - e^{-t/\tau}\right)$$

- time-dependent case: $\alpha_\tau = 1$

$$N_E(t) = \int_{t_0}^{t} d\log t' \;\; t' Q_E(t') \, G(t, t') = Q_E(t)\tau_{adi}(t) \; \frac{1}{\alpha_Q + \alpha_{adi} + 1} \frac{9-w}{3(5-w)} \cdot \left(1 - \left(\frac{t}{t_0}\right)^{-\alpha_Q - \alpha_{adi} + 1}\right)$$

≈ 0.7

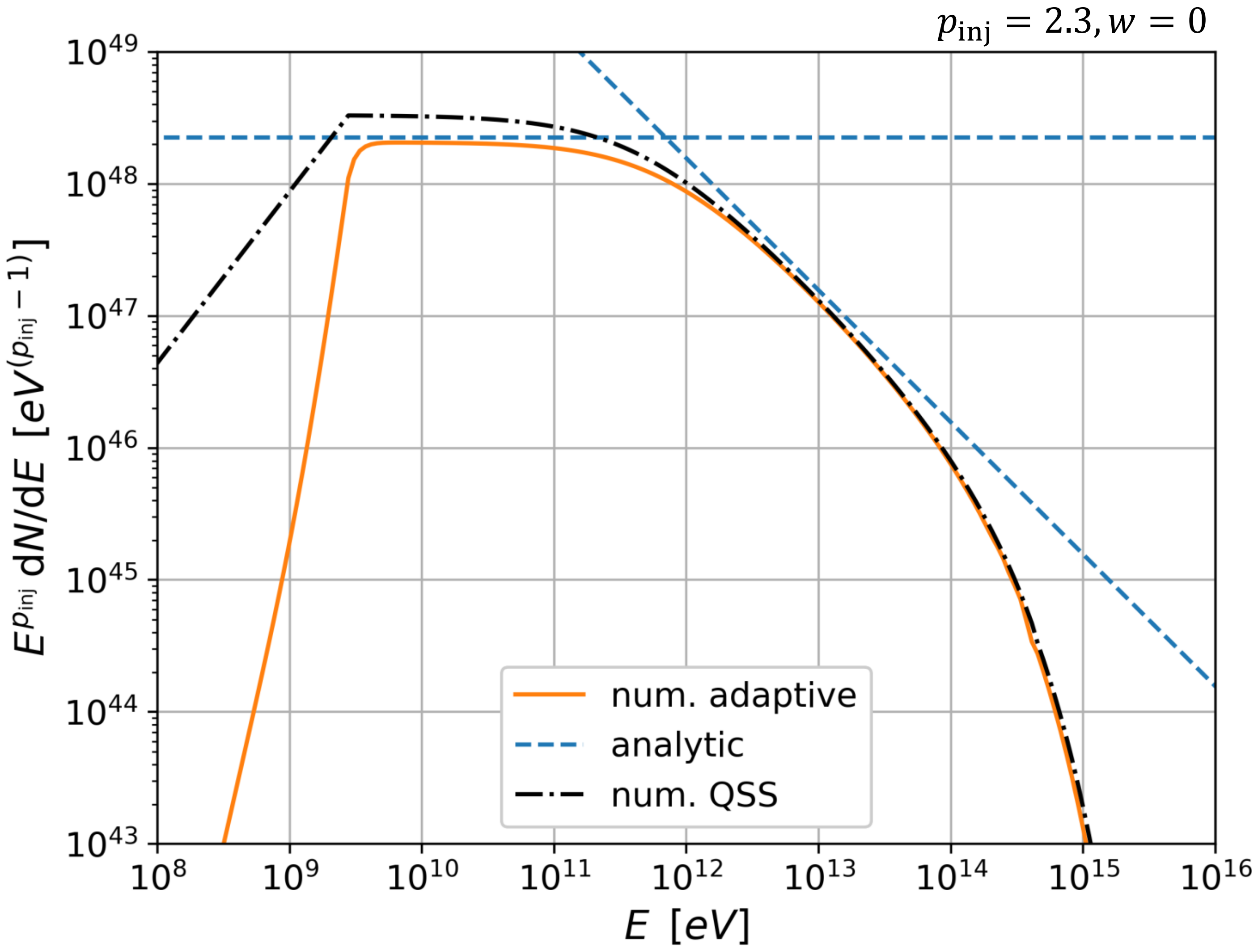
numerical factor $\mathcal{O}(1)$ convergence term

$\propto t'^{\alpha_Q}$ $\propto t'^{-\alpha_{adi}}$

$$\alpha_Q = -\frac{2 + (p-2)(3-w)}{5-w} \approx -0.4 \qquad \alpha_{adi} = (p-1)\frac{9-w}{3(5-w)} \approx 0.7$$

quasi steady state
(steady state with instantaneous parameters)

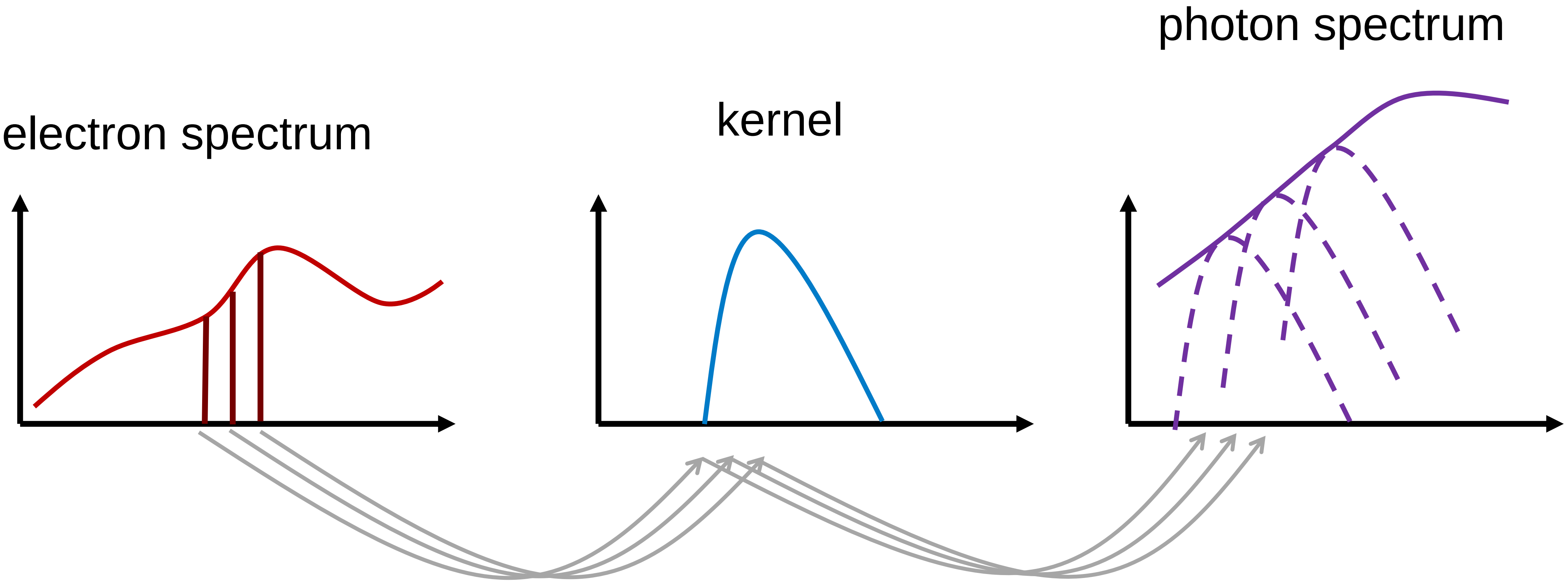
Effective Electron Spectrum



→ smoothly broken power law!

Radiation processes: SSC

- just another example of convolutions



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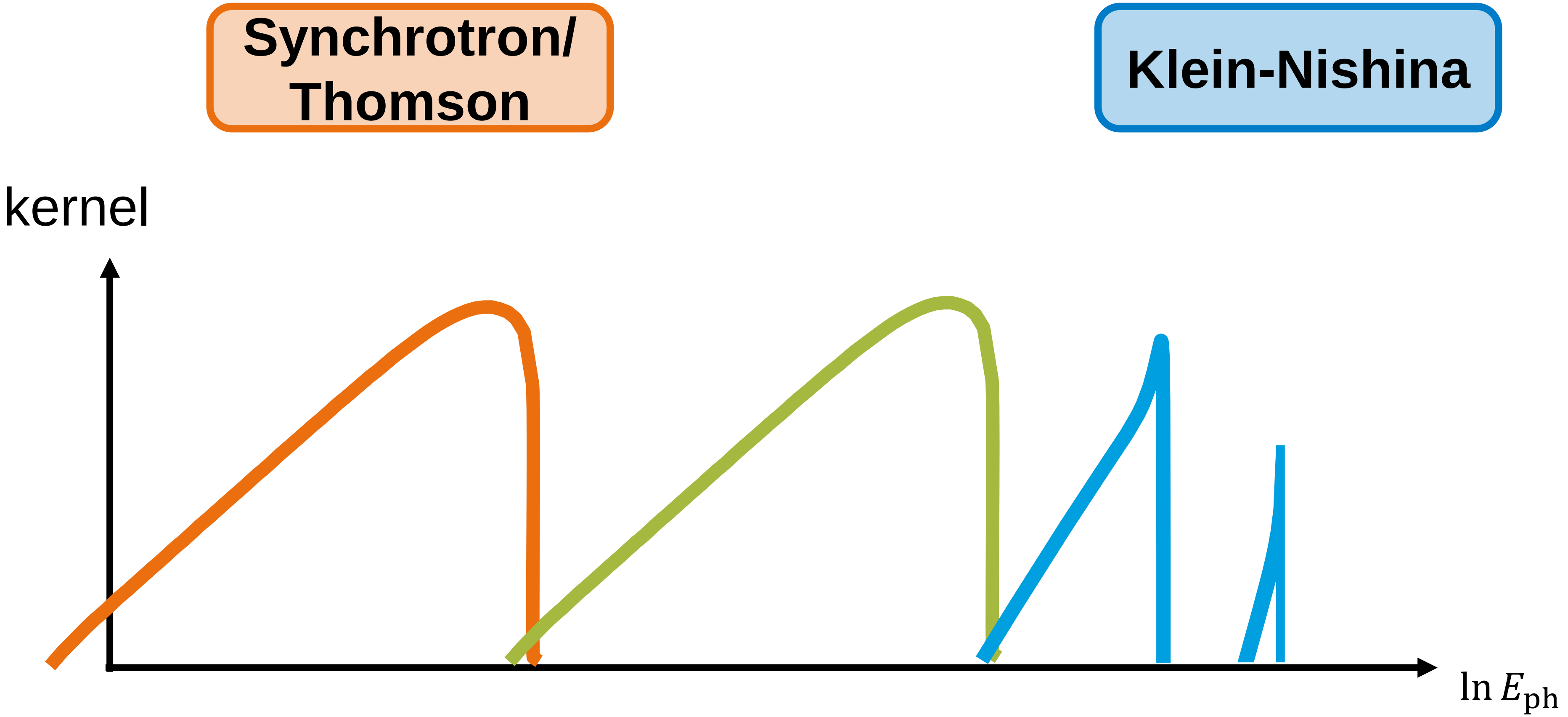

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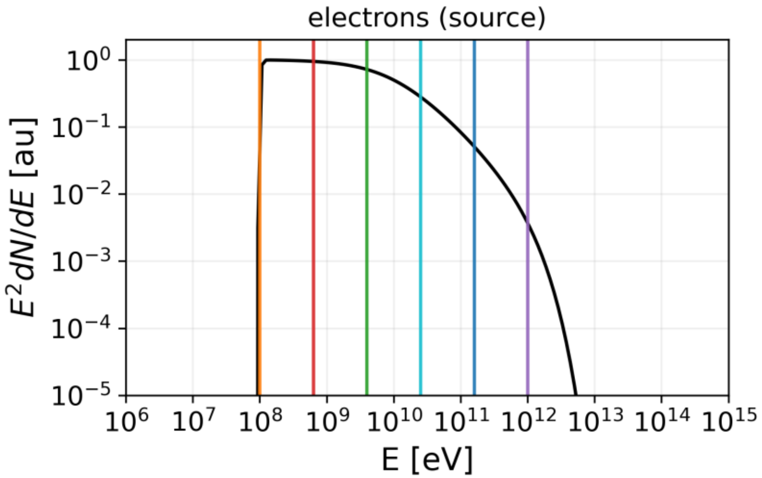
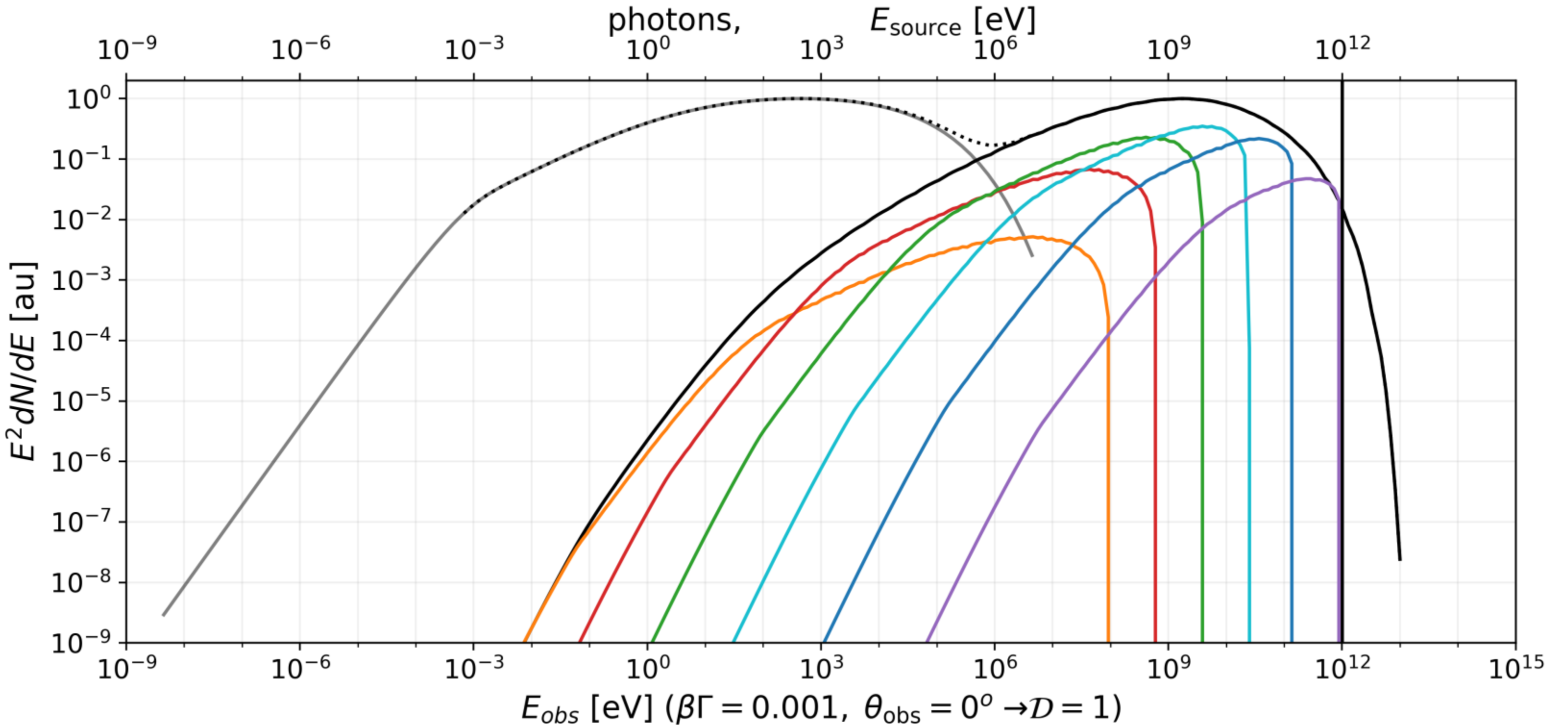


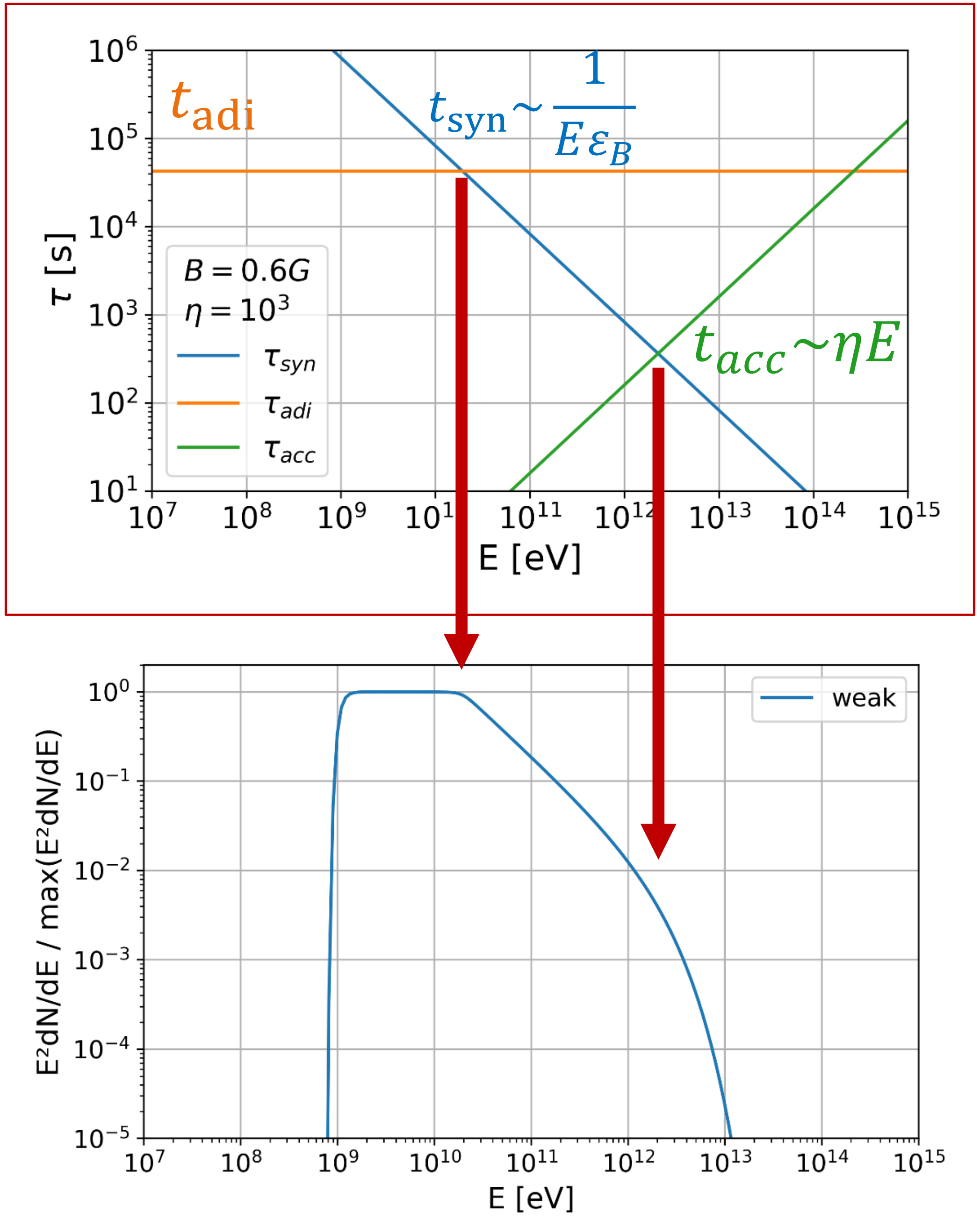


Reduced SSC Model

- smoothly broken PL electrons from quasi steady state ($N \sim Q\tau$)

  $\rightarrow$ break from magnetic field ε_B

  $\rightarrow$ maximum energy set via η



Reduced SSC Model

- smoothly broken PL electrons from quasi steady state ($N \sim Q\tau$)

  → break from magnetic field ε_B

  → maximum energy set via η

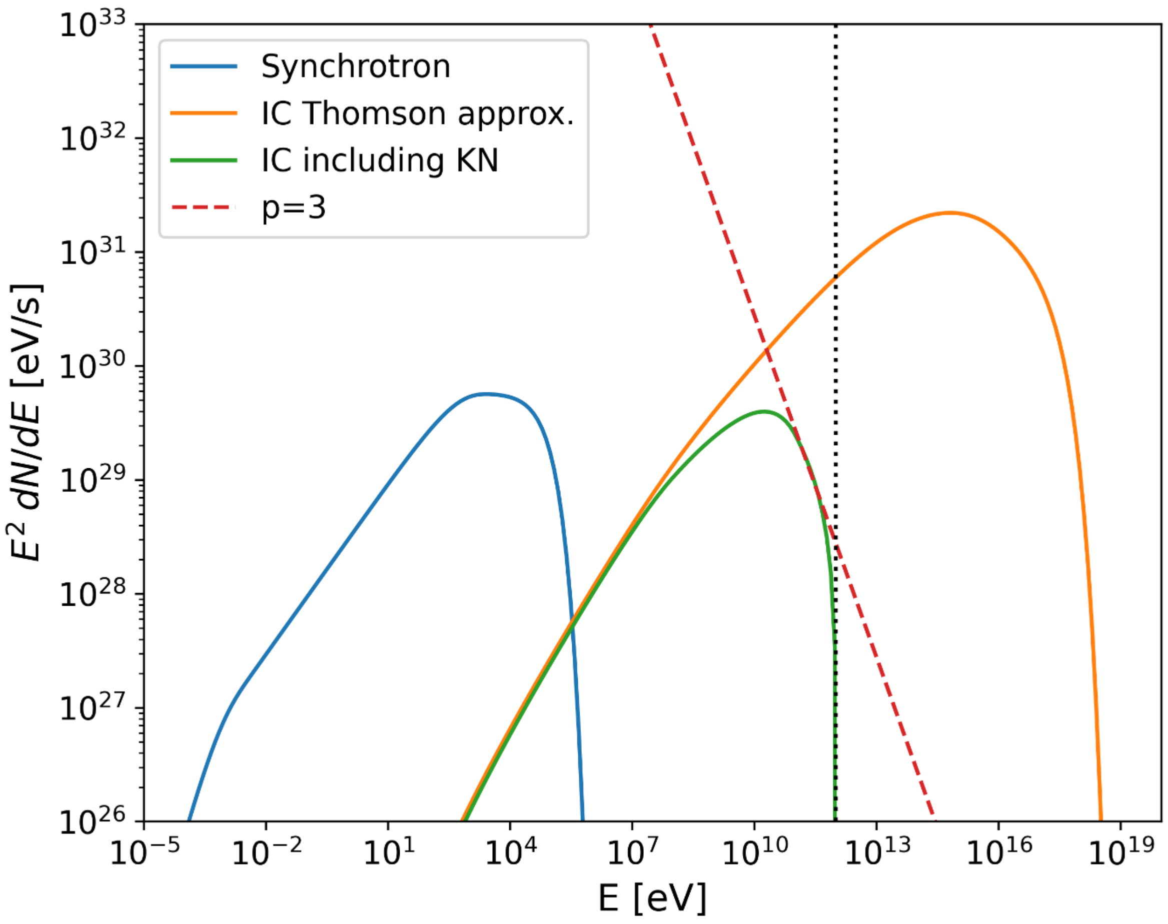
- photon spectrum of these electrons

  → Synchrotron component

  → SSC component

→ Andrew's talk, …

→ MWL fitting on Wednesday



Summary

- relativistic shock partitions pressure/energy into fractions ε_B, ε_e
- even in time dependent modeling, the smoothly broken power law is a reasonable effective description of the electron spectrum

  → quasi-steady state

  → reduced SSC model focusses modeling to essence

Where this picture is very simple

- homogeneous box of width Δ

  $\rightarrow$ blast wave has a profile

- magnetic field strength distribution

  $\rightarrow$ only δ-like strength

- injection

  $\rightarrow$ spatially homogeneous

  $\rightarrow$ power law with exponential cutoff, what about thermal particles?

- pair-production?

- jet structure? Viewing angle?